

Math 255 - Spring 2022
Essentials of congruence proofs
10 points

This homework invites you to prove essential facts about congruences that we will use throughout the semester. The proofs should follow somewhat directly from the definition of congruence; the last problem might force you to play with numbers a bit. In this homework, a, b, c, d, n and m are all positive integers.

1. If $a \equiv b \pmod{n}$ and $m|n$, then $a \equiv b \pmod{m}$.
2. If $a \equiv b \pmod{n}$ and $c > 0$, then $ca \equiv cb \pmod{cn}$.
3. If $a \equiv b \pmod{n}$ and the integers a, b, n are all divisible by $d > 0$, then

$$\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{n}{d}}.$$

4. If $a \equiv b \pmod{n}$, show that $\gcd(a, n) = \gcd(b, n)$.
5. (a) Prove or disprove: If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.
(b) Prove or disprove: If $a^2 \equiv b^2 \pmod{n}$, then $a \equiv b \pmod{n}$ or $a \equiv -b \pmod{n}$.