Math 395 - Spring 2020
Midterm Exam

Please solve **BOTH** problems below:

1. Let R be a commutative ring with 1 and let A, B and C be left R-modules. Prove that $\operatorname{Hom}_R(A, B \oplus C) \cong \operatorname{Hom}_R(A, B) \oplus \operatorname{Hom}_R(A, C)$, where this is an isomorphism of R-modules.

2. Let X be any nonempty set and let R be the (commutative) ring of all integer-valued functions on X under the usual pointwise operations of addition and multiplication of functions:
$$R = \{f \mid f\colon X \to \mathbb{Z}\}.$$
For each $a \in X$, define
$$M_a = \{f \in R \mid f(a) = 0\}.$$
   (a) Prove that M_a is a prime ideal in R.
   (b) Prove that M_a is not a maximal ideal in R.
   (c) Find all units in R.
   (d) Find all zero divisors in R.