Name:

Problem 1: Consider the function $h: \mathbb{R} \to \mathbb{R}$ given by

$$h(x) = \begin{cases} 2 & \text{if } x \ge 0, \\ -2 & \text{if } x < 0, \end{cases}$$

where here \mathbb{R} is given the standard topology. Prove that h is not continuous.

Solution: Consider the open set $(1,3) \subset \mathbb{R}$. Then $h^{-1}((1,3)) = [0,\infty)$, which is not open. Therefore h is not continuous.