Math 295 - Spring 2020 Homework 3

This homework is due on Wednesday, January 29. All problems are adapted from Munkres's *Topology*.

- 1. Determine which of the following statements are true for all sets A, B, and C. If an equality fails, determine whether the statement becomes true if the "equal" symbol is replaced by one or the other of the inclusion symbols \subset or \supset .
 - (a) A (A B) = A B
 - (b) $A \cap (B C) = (A \cap B) (A \cap C)$
 - (c) $A \cup (B C) = (A \cup B) (A \cup C)$
- 2. Let X be a topological space and A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.
- 3. Show that the collection \mathcal{T}_c given in Example 4 of Section 12 is a topology on the set X.
- 4. Let X be any set. Is the collection

$$\mathcal{T}_{\infty} = \{U \mid X - U \text{ is infinite or empty or all of } X\}$$

a topology on X?