Name:

Problem 1: *Lemma 3 of Section 6 states that if p is an odd prime, then the least residues*

$$2, 3, 4, \ldots, p-4, p-3, p-2$$

can be partitioned into $\frac{p-3}{2}$ pairs (a, a') such that for each pair,

$$aa' \equiv 1 \pmod{p},$$

with $a \not\equiv a' \pmod{p}$.

Let $p = 11$. Partition the set

$$\{2, 3, 4, 5, 6, 7, 8, 9\}$$

into four pairs (a, a') such that in each case $aa' \equiv 1 \pmod{11}$.

Solution:
We have that $12 \equiv 1 \pmod{11}$, therefore we get

$$2 \cdot 6 = 12 \equiv 1 \pmod{11} \quad \text{and}$$
$$3 \cdot 4 = 12 \equiv 1 \pmod{11},$$

which gives us the two pairs $(2, 6)$ and $(3, 4)$.
We can also negate each integer:

$$(-2) \cdot (-6) \equiv 9 \cdot 5 \equiv 1 \pmod{11} \quad \text{and}$$
$$(-3) \cdot (-4) \equiv 8 \cdot 7 \equiv 1 \pmod{11},$$

which gives us the pairs $(5, 9)$ and $(7, 8)$.
All of the least residue classes now belong to one pair, and we are done, the pairs are

$$(2, 6), (3, 4), (5, 9), (7, 8).$$