Math 255: Spring 2018
Practice Exam 2

NAME:

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Problem	Value	Score
1	12	
2	6	
3	8	
4	8	
5	8	
6	8	
GC	8	
TOTAL	50 (or 55)	

Problem 1 : (12 points) Solve the following equations. For each equation, give **all** distinct solutions (if there are more than one) and be sure to clearly indicate which ring the solutions belong to.

a) $4x \equiv 6 \pmod{18}$

b) $3x \equiv 2 \pmod{19}$

c) $9x \equiv 7 \pmod{15}$

Problem 2 : (6 points) Solve the following system of equations. Be sure to give **all** distinct solutions (if there are more than one) and to clearly indicate which ring the solution(s) belong to.

$$6x \equiv 6 \pmod{24}, \quad 3x \equiv 6 \pmod{9}, \quad 9x \equiv 7 \pmod{14}$$

Problem 3 : (8 points) If p is a prime, show that for any integer a,

$$a^p + (p-1)!a \equiv 0 \pmod{p}.$$

Problem 4 : (8 points) Find the remainder when 15! is divided by 17.

Problem 5 : (8 points) Show that $\sigma(n)$ is odd if and only if n is either a perfect square or twice a perfect square.

Problem 6 : (8 points) Let $\omega(1) = 0$ and, for $n > 1$ let $\omega(n)$ denote the number of distinct prime divisors of n. In other words, if $n = p_1^{e_1} \dots p_k^{e_k}$ is prime-power decomposition of n, then $\omega(n) = k$.

a) Give the definition of a multiplicative function.

b) Prove that $f(n) = 2^{\omega(n)}$ is multiplicative.

Extra problem for graduate credit:

Problem 7 : (8 points) Let p be a prime of the form $p = 1 + 4k$. Show that

$$\left(\left(\frac{p-1}{2}\right)!\right)^2 \equiv -1 \pmod{p}.$$