Math 255 - Spring 2018
Homework 9

This homework is due on Monday, April 2.

1. Consider the system of congruences

$$6x \equiv 3 \pmod 9$$
$$10x \equiv 8 \pmod{16}.$$

   (a) Solve each of the following congruences:
      i. $6x \equiv 3 \pmod 9$
      ii. $10x \equiv 8 \pmod{16}$
   (b) The system of congruences above is equivalent to 6 distinct systems of congruences of the form $x \equiv a_1 \pmod 9$, $x \equiv a_2 \pmod{16}$. Write down these 6 systems and solve each of them using the Chinese Remainder Theorem.
   (c) The system of congruences above is equivalent to a single system of congruences of the form $a_1x \equiv b_1 \pmod 3$, $a_2x \equiv b_2 \pmod 8$. Write down this system, solve it using the Chinese Remainder Theorem, and lift your solutions to $\mathbb{Z}/144\mathbb{Z}$.

2. Solve each of the following systems of congruences. For each system of equations, be sure to list **all** distinct solutions, and the ring they belong to.
   (a) $4x \equiv 4 \pmod 8$, $5x \equiv 6 \pmod{25}$, $3x \equiv 6 \pmod{27}$
   (b) $2x \equiv 6 \pmod 8$, $2x \equiv 8 \pmod 9$, $3x \equiv 3 \pmod{18}$

3. Find a multiple of 7 that leaves a remainder of 1 when divided by $2, 3, 4, 5$ and 6.

4. Let x, r, s and m be integers, with $m > 1$. Show that if $x \equiv r \pmod m$ and $x \equiv s \pmod{m+1}$, then

$$x \equiv r(m+1) - sm \pmod{m(m+1)}.$$

Extra problem for graduate credit:

5. Notice that the three consecutive integers 48, 49 and 50 each have a square factor. In this problem we will investigate when/if this happens.
   (a) Give an expression for all integers n such that $3^2|n$, $4^2|(n+1)$ and $5^2|(n+2)$.
   (b) Is there n such that $2^2|n$, $3^2|(n+1)$ and $4^2|(n+2)$?