

Name:

Problem 1: *Consider the equation*

$$ax + by = c,$$

where $a, b, c \in \mathbb{Z}$. Theorem 2.9 in Section 2.5 of the book states when this equation has integer solutions (alternatively, when we want integer solutions we might say we are solving a Diophantine equation).

Use this theorem to determine if the equation

$$2x + 6y = 5$$

has any integer solutions. Explain neatly but briefly how you are applying the theorem to obtain your conclusion.

Solution: The theorem says that

$$ax + by = c$$

has integer solutions if and only if the greatest common divisor of a and b divides c .

In this particular instance, we have $a = 2$, $b = 6$ and $c = 5$. We have that $\gcd(2, 6) = 2$. However, 2 does not divide 5. Therefore there is no integer solution to the equation.