MATH 255 QUIZ 20 APRIL 10 2017

Name:

Problem 1: *It is a fact that* 2 *is a primitive root of* 5*. Here is a table of discrete logarithms in base* 2 *modulo* 5*:*

a	1	2	3	4
$\log_2 a$	0	1	3	2

Use this table to give all solutions to the equation

$$3x^{13} \equiv 4 \pmod 5.$$

Solution:
We have that $3 \equiv 2^3 \pmod 5$ and $4 \equiv 2^2 \pmod 5$. If $x \equiv 2^k \pmod 5$, then $x^{13} \equiv 2^{13k} \pmod 5$. Plugging this all in, the equation becomes

$$2^3 2^{13k} \equiv 2^2 \pmod 5$$

or

$$2^{13k+3} \equiv 2^2 \pmod 5.$$

Taking $\log_2$ on both sides we get

$$13k + 3 \equiv 2 \pmod 4.$$

We can reduce this more and subtract 3 from both sides to get

$$k \equiv -1 \pmod 4$$

or

$$k \equiv 3 \pmod 4.$$

Therefore there is a unique solution and it is $x \equiv 2^3 \equiv 3 \pmod 5$.