

Math 395 - Fall 2021
Beginner Reading 3

This reading is “due” on Monday September 20 at 11:59pm.

This week you are invited to read Chapter 3 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 3.1

1. Read the example on page 74 of the textbook, and for now let's set $n = 3$. How is Z_3 (which I usually denote by C_3) different from the set of fibers of $\phi: \mathbb{Z} \rightarrow Z_3$? How are they the same?
2. What is your favorite result from Proposition 1?
3. Read Proposition 2, part 2. If you recast this situation as having the subgroup K acting on G by left multiplication, then what would you call X in group action vocabulary? What is the name given to X in the definition following Proposition 2?
4. What is the **trivial** homomorphism? Why do you think it's called that?
5. Read Proposition 5 carefully. Given a group G and a subgroup N , can we always define a multiplication on the cosets of N by multiplying representatives?
6. True or false: All subgroups of a cyclic group are cyclic, and all quotients of a cyclic group are cyclic.

Section 3.2

7. Let G be a group and H be a subgroup of G . What is the index of H in G and how do we denote it?
8. Let p be a prime and let G be a group of order p . What can you say about G ?
9. What is a simple group?
10. State the converse of Lagrange's Theorem. Is it true? What partial converses of Lagrange's Theorem are true?
11. Let G be a group and H and K be subgroups of G . Is HK a subgroup of G ?

Section 3.3

12. Copy down the four isomorphism theorems for groups.

13. Does knowledge of the isomorphism classes of G/N and N give knowledge of the isomorphism class of G ?
14. What does it mean to say that the homomorphism Φ **factors through** a subgroup N ?

Section 3.4

15. What is a composition series? Why is the composition series of a group important?
16. The definition of solvable presents a chain of subgroups of a group G . Is this chain of subgroups a composition series?

Section 3.5

17. True or false: There is a unique way to write every element of S_n as a product of cycles, even if we don't force them to be disjoint.
18. What is a transposition?
19. True or false: Every element of S_n can be written as a product of transpositions.
20. What is the sign of a permutation? What do we call this permutation if the sign is $+1$? What do we call this permutation if the sign is -1 ?
21. What is the alternating group? What is its order?
22. Is a 5-cycle even or odd?
23. True or false: If the order of a permutation is odd, then it is an odd permutation.
24. True or false: If the order of a permutation is even, then it is an even permutation.