

Math 395 - Fall 2021
Beginner Reading 2

This reading is “due” on Monday September 13 at 11:59pm.

This week you are invited to read Chapter 2 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 2.1

1. What does the subgroup criterion say?
2. Apply the subgroup criterion to show that $3\mathbb{Z}$, the set of integers that are a multiple of 3, is a subgroup of \mathbb{Z} under addition.

Section 2.2

3. Show that for any subset A of a group G , the normalizer $N_G(A)$ is a subgroup of G , following steps similar to the proof that $C_G(A)$ is a subgroup of G .
4. What does Lagrange’s Theorem say?
5. Prove that if G acts on a set A , the kernel of the action is a subgroup of G .

Section 2.3

6. Let C_{10} be the cyclic group with 10 elements (this is denoted Z_{10} in Dummit and Foote). How many generators does C_{10} have?
7. We have that $C_{10} \cong \mathbb{Z}/10\mathbb{Z}$, where $\mathbb{Z}/10\mathbb{Z}$ is a group under addition. Which specific elements of $\mathbb{Z}/10\mathbb{Z}$ generate $\mathbb{Z}/10\mathbb{Z}$?
8. List all of the subgroups of $\mathbb{Z}/10\mathbb{Z}$.

Section 2.4

9. Let $A = \{a, b\}$. Write three words in A .
10. How many isomorphism classes of groups of order 6 are there in total?

Section 2.5

11. Write the lattice of subgroups of $\mathbb{Z}/10\mathbb{Z}$. (You can use your work from Problem 8 to get started.)