Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

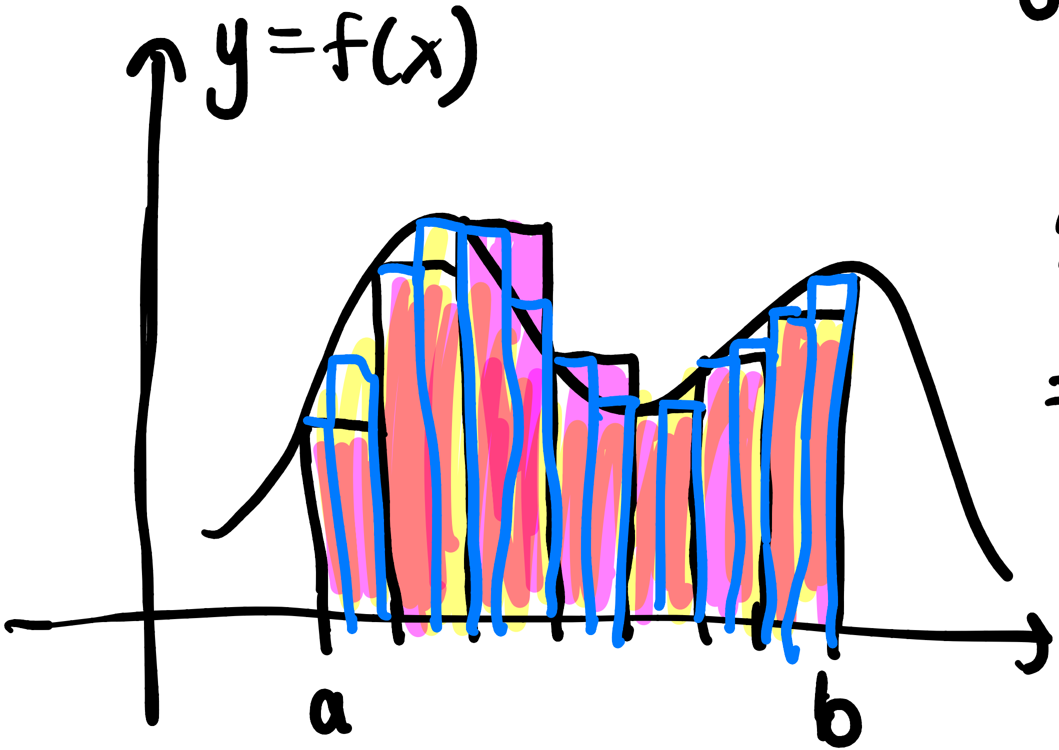
Mental health week:

- no warm ups this week (not Wed either)

  +2 warm ups for everybody

- there will be a short HW
- you can turn in a metacognition essay

Remember from calculus



$\int_a^b f(x)\,dx$

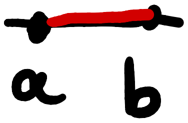
= area = ??

≈ area under

= area of the rectangles as width → 0

We will not cover the limit definition of integrals, leave this to Math 241

Notation recall real analysis $\int_a^b f(x)dx$



no choice of path from a to b

But in complex analysis there is so

$\int_a^b f(z)dz$ doesn't make sense in general

But in complex analysis there is so

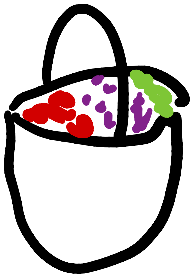
$$\int_a^b f(z)\,dz$$ doesn't make sense in general

because the integral might depend on path that we take from a to b

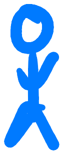
So we will write

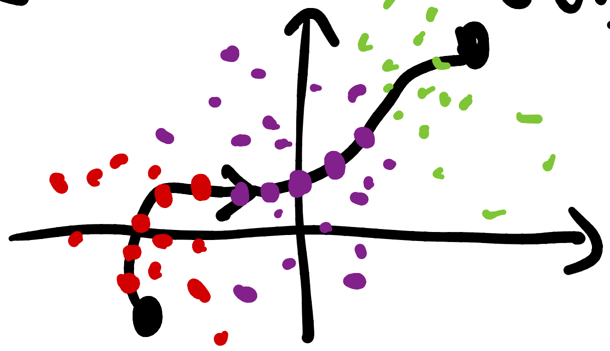
$$\int_\gamma f(z)\,dz$$ where γ is a contour/path

$\gamma : [0,1] \to \mathbb{C}$

$$\int_\gamma f(z)\,dz =$$ 

$\gamma(1) = b$





$\gamma(0) = a$

where the person walks

- where they start, end, where they go in the middle

2 integral rules

(1) $\int_\gamma c \cdot f(z)\,dz = c \cdot \int_\gamma f(z)\,dz$

ex: $\int_\gamma 2z\,dz = \underline{2} \int_\gamma z\,dz$

constant

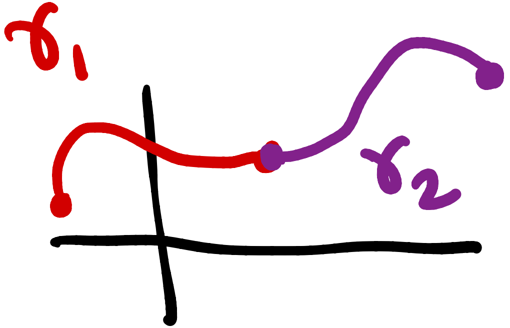
(2) $\int_\gamma (f_1(z) + f_2(z))\,dz = \int_\gamma f_1(z)\,dz + \int_\gamma f_2(z)\,dz$

ex $\int_\gamma (z^2 + z)\,dz = \int_\gamma z^2\,dz + \int_\gamma z\,dz$

② $$\int_\gamma \big(\underbrace{f_1(z)}_{\text{flowers}}+\underbrace{f_2(z)}_{\text{veggies}}\big)\,dz = \int_\gamma \underbrace{f_1(z)}_{\text{flowers}}\,dz + \int_\gamma \underbrace{f_2(z)}_{\text{veggies}}\,dz$$

ex $$\int_\gamma (z^2+z)\,dz = \int_\gamma z^2\,dz + \int_\gamma z\,dz$$

③ $$\int_{\gamma_1+\gamma_2} f(z)\,dz = \int_{\gamma_1} f(z)\,dz + \int_{\gamma_2} f(z)\,dz$$

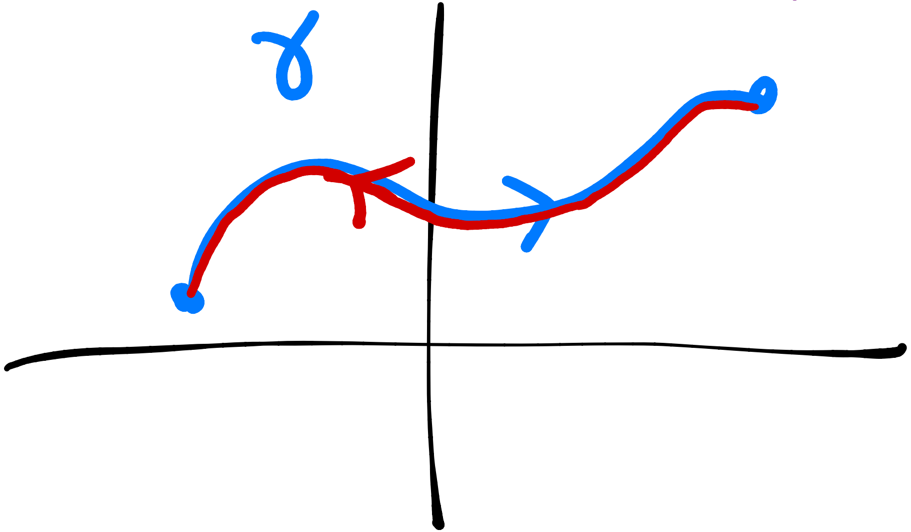


④ $$\int_{-\gamma} f(z)\,dz = -\int_{\gamma} f(z)\,dz$$

backwards

negative answer

watch the video in reverse



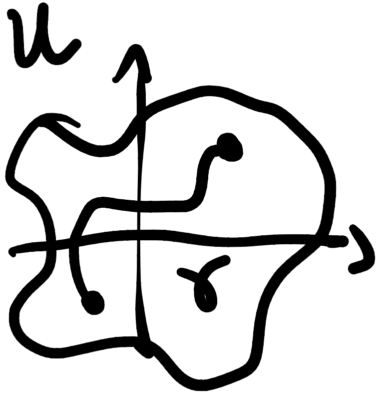
Definition

Let U be a region in $\mathbb{C}$ $f: U \to \mathbb{C}$ continuous

$\gamma: [a,b] \to \mathbb{C}$ a contour in U. Then we

define

$$\int_\gamma f(z)\,dz = \int_a^b f(\gamma(t))\,\gamma'(t)\,dt$$

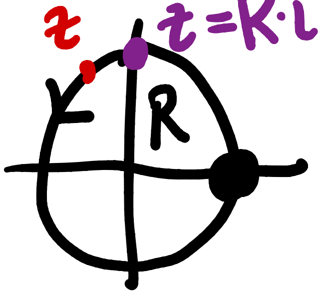


U open, connected

Task 167

$R > 0 \quad \gamma(t) = Re^{it} \quad 0 \le t \le 2\pi$

$$\gamma'(t) = Re^{it} \cdot i$$



$t = \frac{\pi}{2}$

on path

$z = \gamma(t)$

$$\int_\gamma z^2 \, dz$$

$$= \int_0^{2\pi} (\underbrace{Re^{it}}_{\gamma(t)})^2 \underbrace{Rie^{it}}_{\gamma'(t)} \underbrace{dt}_{dt}$$

we'll finish on Wednesday

THAT'S ALL FOR TODAY!

OH 12pm-1pm