Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Grading: Nothing is graded.

Rest of class: Questions

① Why open sets, connected, bounded?
Paths?

HW2 #2a) iii

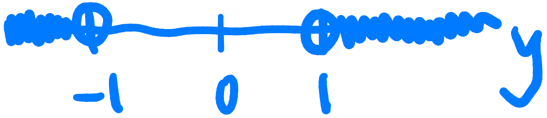
$$\{z \in \mathbb{C} : |\mathrm{Im}(z)| > 1\}$$

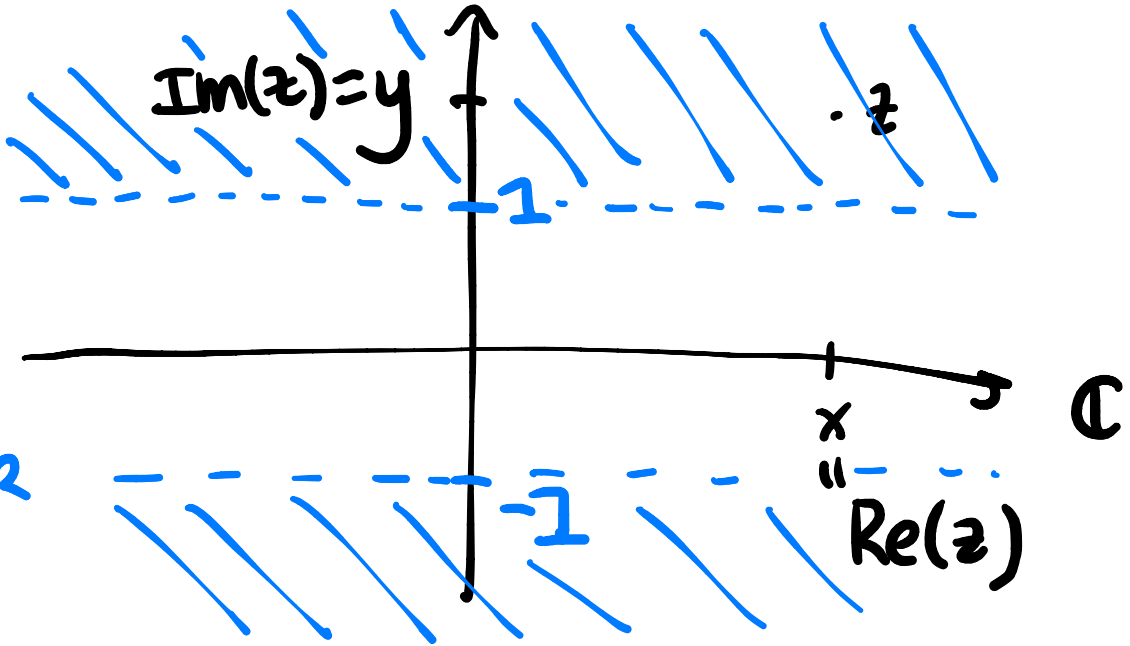
usual absolute value $\mathrm{Im}(z) \in \mathbb{R}$

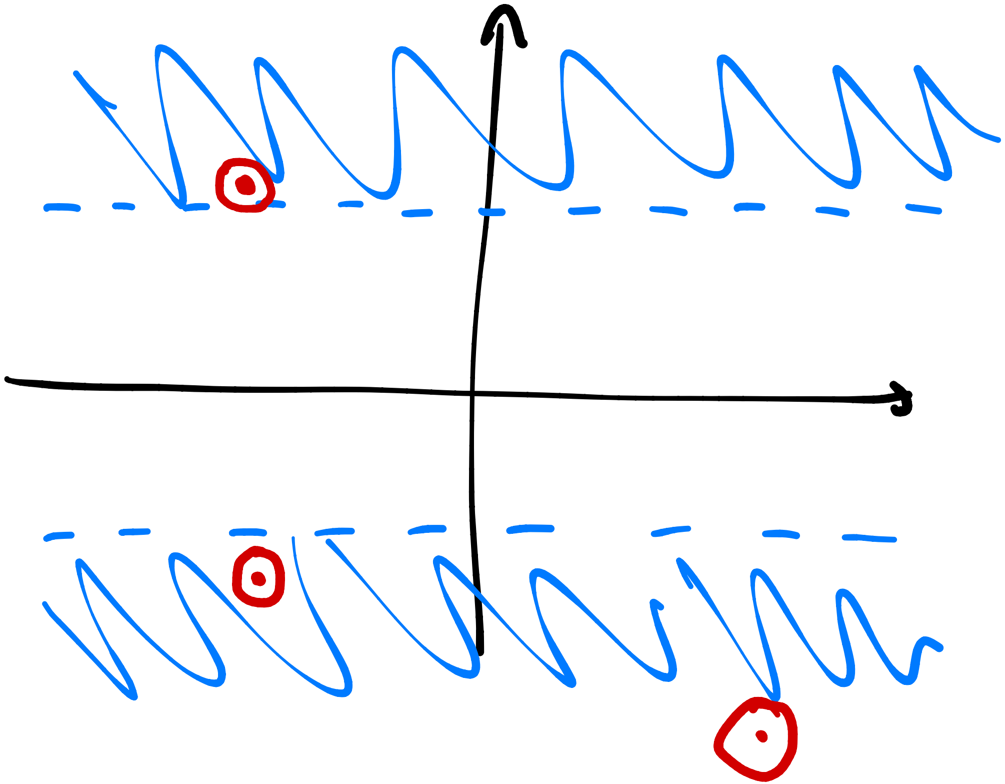
$z = x + iy$

$|y| > 1$

$\iff y < -1$ OR $y > 1$







$\{z \in \mathbb{C} : |\mathrm{Im}(z)| > 1\}$

#4 a) this set is open
not closed
every pt has a little ball around it that is in the set

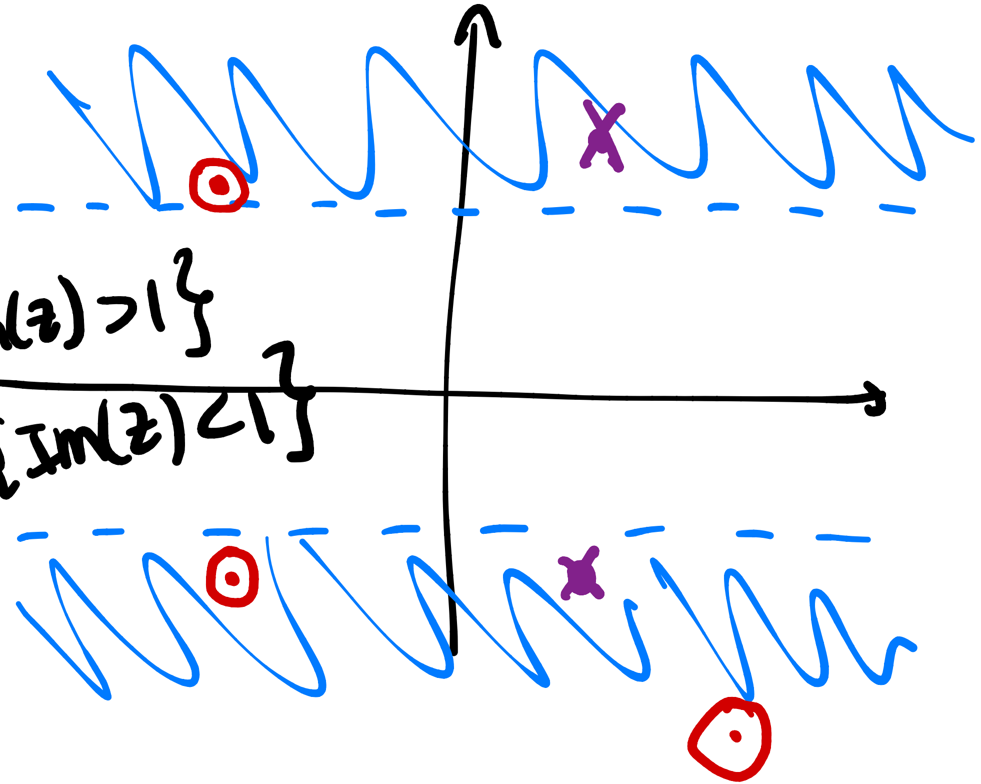
b) this set is not bounded
it is not contained in any ball

$|\mathrm{Im}(z)| > 1$

$U = A \cup B$

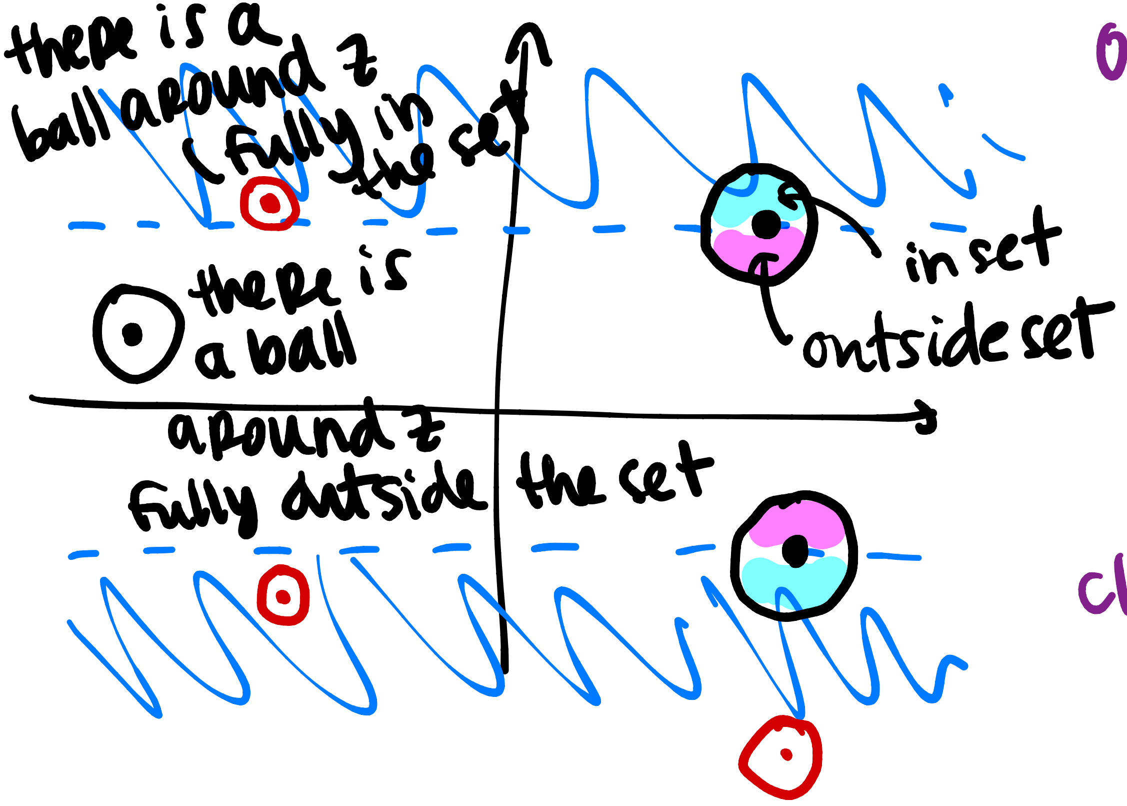
$A = \text{top} = \{\mathrm{Im}(z) > 1\}$

$B = \text{bottom} = \{\mathrm{Im}(z) < 1\}$



c) this set is not connected   the top & bottom parts cannot be joined

d) boundary:



open: all the points are interior points (have a ball around them that is fully in the set)

closed: contains all its boundary points

Boundary point: every ball around z touches points inside and outside the set

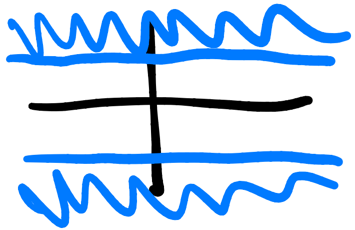
Boundary points $\{z \in \mathbb{C} : \operatorname{Im} z = 1\}$

$$\cup \{z \in \mathbb{C} : \operatorname{Im} z = -1\}$$

$$= \{z \in \mathbb{C} : |\operatorname{Im}(z)| = 1\}$$

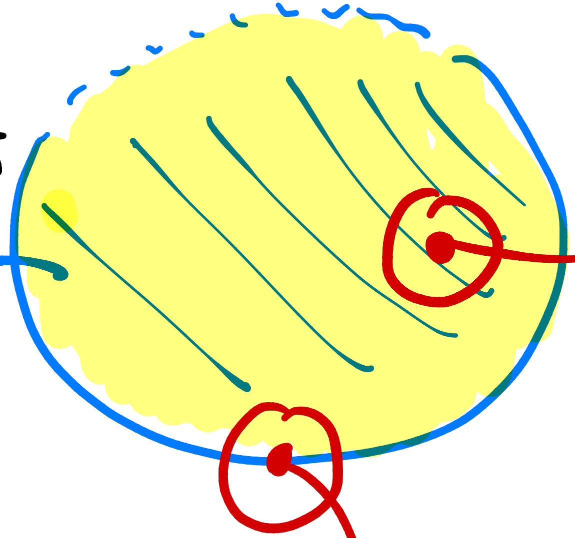
Interior points are the elements of the set itself.

Closure of this set (take your set + add the boundary points, so its closed)



An open set contains only interior points (no boundary)

E



always in E

interior

∃ a ball (can be small) that's fully inside E

boundary — inside E or not

∀ balls, the ball has points in E and points outside of E

A closed set contains all the boundary points

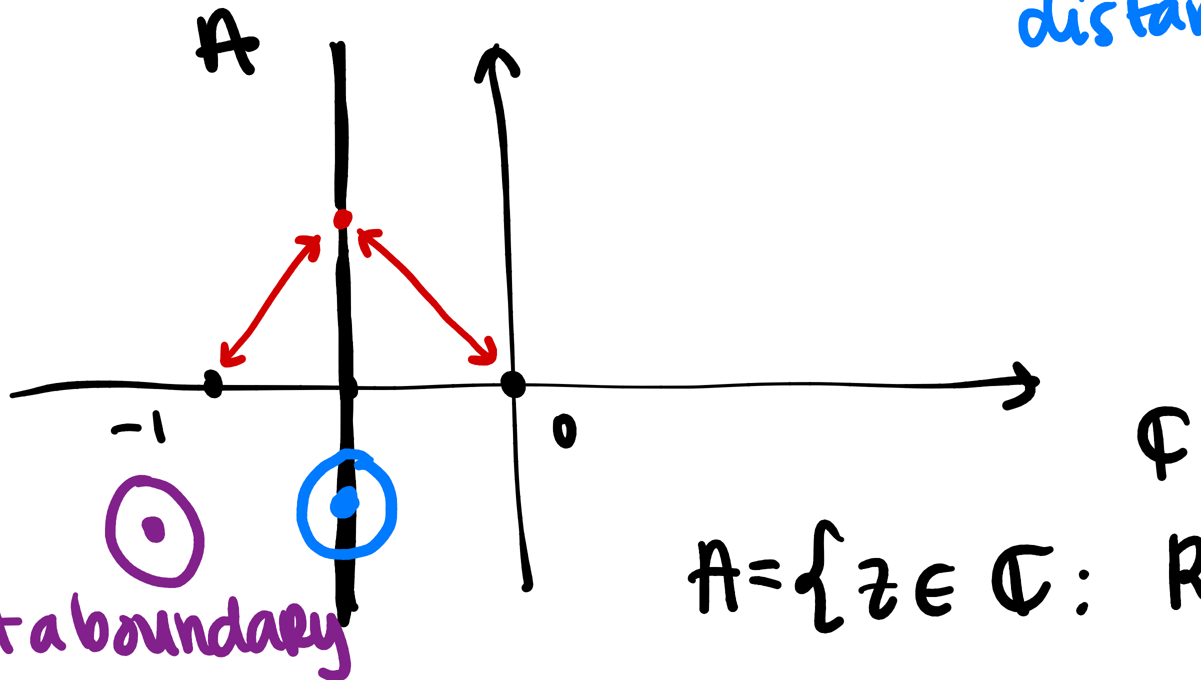
HW 2 #2a) ii $\{z\in\mathbb{C}: \underbrace{|z|}_{\text{distance between 0 and } z} = \underbrace{|z+1|}_{\text{distance between } -1 \text{ and } z}\} = A$

$d(z_1,z_2)=|z_1-z_2|$

distance between 0 and z

distance between -1 and z

$|z-(-1)|$



$A=\{z\in\mathbb{C}: \operatorname{Re}(z)=-\frac{1}{2}\}$

Can also do this algebraically

$$|z| = \sqrt{x^2+y^2}$$
$$|z+1| = \sqrt{(x+1)^2+y^2}$$

solve

$$\sqrt{x^2+y^2} = \sqrt{(x+1)^2+y^2}$$

...

$$x = -\frac{1}{2}$$

#4 d) interior points? no, none

boundary points: every point of A is a boundary point

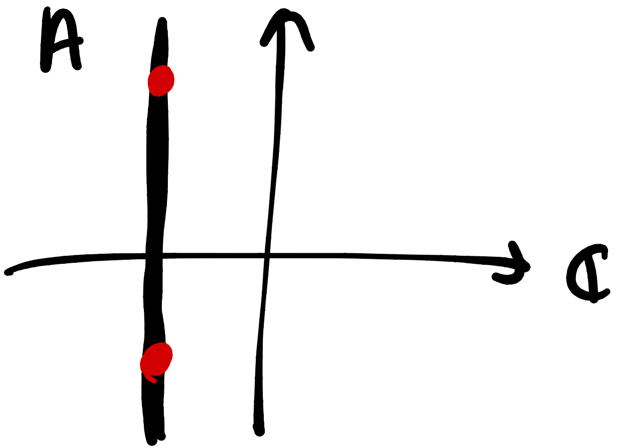
a) this set is closed (it contains its whole boundary)

this set is not open (the points it contains are not all interior points)

(it contains points that are not interior points)

b) not bounded

c) connected



Next week: derivatives!

$f: U \to \mathbb{C} \qquad U \subseteq \mathbb{C}$

$z - z_0 = h$

$z = z_0 + h$

f is differentiable at z_0 if

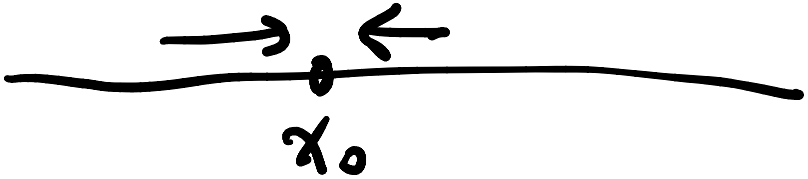
$$\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \text{ exists}$$

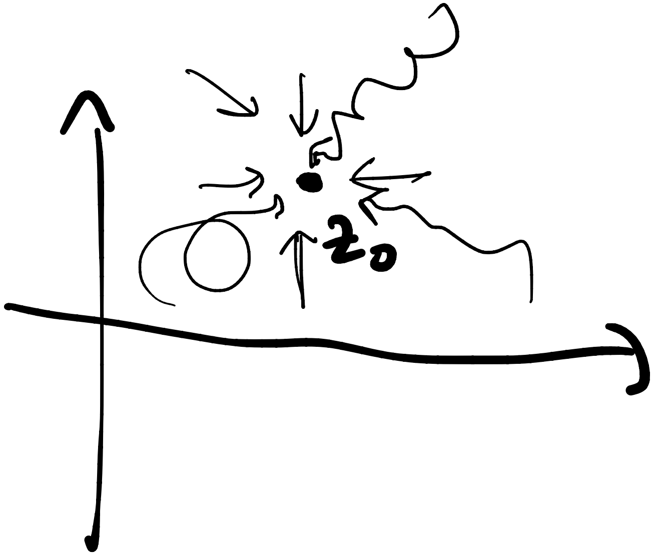
$$= \lim_{h \to 0} \frac{f(z_0 + h) - f(z_0)}{h}$$

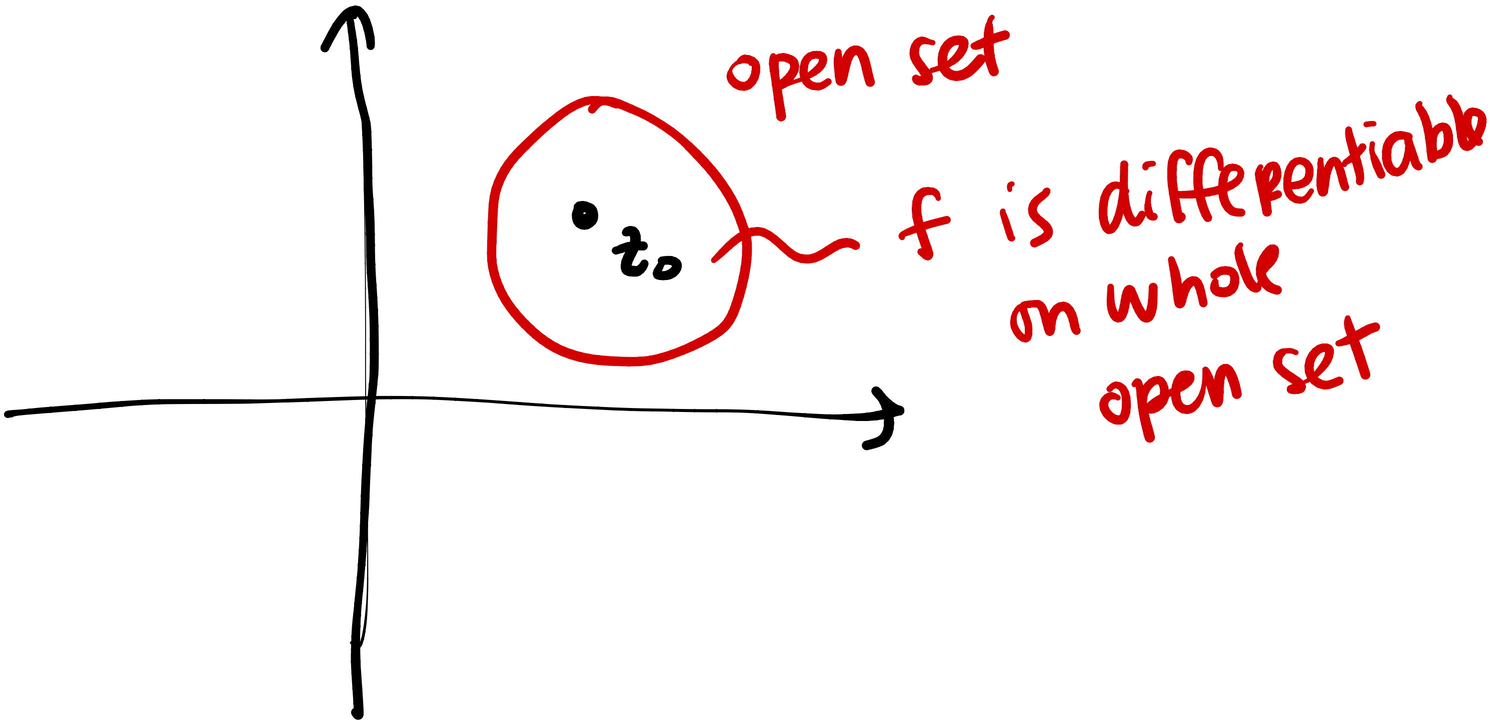
This is a big deal because now $h \in \mathbb{C}$

$$\lim_{h \to 0} \frac{f(z_0+h) - f(z_0)}{h}$$

$\mathbb{R}$ derivative from left + right






open set
z_0
f is differentiable on whole open set

That's all for today!