Name:

Problem 1: *Consider the rational function given by the following rule:*

$$f(x) = \frac{x+1}{x^2 - x - 2}.$$

a) What is the domain of this function?

The domain of this function is all values of x, except those that make the denominator zero. To find what values make the denominator zero, we solve the equation $x^2 - x - 2 = 0$:

$$\begin{aligned} x^2 - x - 2 &= 0 \\ x^2 - 2x + x - 2 &= 0 \\ x(x-2) + 1(x-2) &= 0 \\ (x+1)(x-2) &= 0. \end{aligned}$$

Therefore the two values that make the denominator zero are $x = -1$ and $x = 2$. The domain is $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$.

b) Does this function have one or many vertical asymptotes? If so, give the equation of each vertical asymptote.

Vertical asymptotes are found among the values of x that make the denominator zero. Therefore the possibilities are $x = -1$ and/or $x = 2$.

We verify if any of these are removed by canceling:

$$f(x) = \frac{x+1}{x^2 - x - 2} = \frac{x+1}{(x+1)(x-2)} = \frac{1}{x-2}.$$

From this, we see that $x = 2$ is a vertical asymptote, but $x = -1$ is not a vertical asymptote.

c) Does this function have one or many holes (or removable discontinuities)? If so, give the x-coordinate of each hole (or removable discontinuity).

Removable discontinuities are found among the values of x that make the denominator zero. We have seen in part b) that after canceling, the singularity at $x = -1$ is removed. Therefore the removable discontinuity is at $x = -1$.