

Name:

Problem 1: *Simplify the following expression completely:*

$$\frac{\sqrt{x\sqrt[3]{y}}}{x^{-2}}.$$

In other words, write this expression as a single positive power of x and a single positive power of y .

Solution: To simplify this expression we must first express the radicals as fractional exponents:

$$\frac{\sqrt{x\sqrt[3]{y}}}{x^{-2}} = \frac{(xy^{1/3})^{1/2}}{x^{-2}}$$

We then apply priority of operations, and get rid of the parentheses. We note that

$$(y^{1/3})^{1/2} = y^{1/6},$$

since when a power is raised to another power, we multiply the exponents, and $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$. We obtain

$$\frac{(xy^{1/3})^{1/2}}{x^{-2}} = \frac{x^{1/2}y^{1/6}}{x^{-2}}$$

We now eliminate the negative exponent using the fact that $\frac{1}{x^{-2}} = x^2$:

$$\frac{x^{1/2}y^{1/6}}{x^{-2}} = x^{1/2}y^{1/6}x^2.$$

Finally, we obtain a single power of x by observing that

$$x^{1/2}x^2 = x^{5/2},$$

since when two exponents with the same base are multiplied, the exponents are added, and $2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$. The final answer is

$$x^{1/2}y^{1/6}x^2 = x^{5/2}y^{1/6}.$$