Name:

Problem 1: *Consider the matrix*

$$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 1 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & 4 \\ -3 & 2 & -2 \end{pmatrix}.$$

a) Write down the columns of the matrix A as separate vectors.

The columns of A are

$$\vec{c}_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \\ -3 \end{pmatrix}, \quad \vec{c}_2 = \begin{pmatrix} -1 \\ 1 \\ -2 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{c}_3 = \begin{pmatrix} 2 \\ 0 \\ -4 \\ 4 \\ -2 \end{pmatrix}$$

b) The column space of A is a subspace of $\mathbb{R}^n$ for some n. What is the value of n?

The column space of A is a subspace of $\mathbb{R}^5$ since all of the vectors that belong to the span of $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$ have 5 entries.

c) Is it possible for the column rank of A to be 4? Why or why not?

That is not possible because the space is spanned by a set of 3 vectors. This is because the column rank is the dimension of the space spanned by the columns, and the dimension is the size of the smallest spanning set possible. Since we have a spanning set with 3 elements, we know that the smallest spanning set will have size 3 or be even smaller. (If the vectors in the spanning set above are not linearly independent, we will be able to "throw some out" and get an even smaller spanning set. Once we have thrown out all of the vectors we can the spanning set will be the smallest possible and be a basis.) Therefore the dimension of this space is at most 3.

d) Write down the rows of the matrix A as separate vectors.

$$\vec{r}_1 = \begin{pmatrix} 2 & -1 & 2 \end{pmatrix}^T, \quad \vec{r}_2 = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix}^T, \quad \vec{r}_3 = \begin{pmatrix} 0 & -2 & -4 \end{pmatrix}^T,$$
$$\vec{r}_4 = \begin{pmatrix} 1 & 1 & 4 \end{pmatrix}^T, \quad \vec{r}_5 = \begin{pmatrix} -3 & 2 & -2 \end{pmatrix}^T$$

e) The row space of A is a subspace of $\mathbb{R}^m$ for some m. What is the value of m?

The row space of A is a subspace of $\mathbb{R}^3$ since all of the vectors that belong to the span of $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, $\vec{r}_4$ and $\vec{r}_5$ have 3 entries.

There is another question on the back.

f) Is it possible for the row rank of A to be 4? Why or why not?

It is not possible for the row rank of A to be 4, because the row space of A is in $\mathbb{R}^3$, which is 3-dimensional. Inside $\mathbb{R}^3$ are only subspaces of dimension 0, 1, 2 and 3.