Name:

Problem 1: *The set*

$$S = \{x^2 - x + 1, 2x + 1, 2x - 1\}$$

is a basis for the vector space $\mathcal{P}_2$ of polynomials of degree less than or equal to 2.

To show this, you would have to check that two conditions are satisfied. Write down what these two conditions are, and what system of linear equations you would solve to check these conditions (including what you would need the outcome to be).

*You **do not** have to solve the equations.*

Solution: The two conditions are: the set is linearly independent and the set spans the whole space.
To show that the set is linearly independent, we solve

$$a_1(x^2 - x + 1) + a_2(2x + 1) + a_3(2x - 1) = 0,$$

and we must show that the only solutions are $a_1 = a_2 = a_3 = 0$.
To show that the set spans the whole space, we must show that any polynomial $ax^2 + bx + c$ can be written as a linear combination of these three polynomials. In other words, we must show that

$$a_1(x^2 - x + 1) + a_2(2x + 1) + a_3(2x - 1) = ax^2 + bx + c$$

always has a solution, no matter what a, b and c are. (Note here that a_1, a_2, a_3 and a are four possibly different values; they do not have to agree.)