DATE: 4/12/2009 # Euler Equations MATH 320

A second-order Euler equation or equidimensional equation is an equation of the form

$$ax^2y'' + bxy' + cy = 0$$

for constants a, b, and c. To solve this equation, we try the solution $y(x) = Ax^r$, which leads us to define the *characteristic equation* of this differential equation

$$ar(r-1) + br + c = 0$$

The solution of the differential equation depends on the roots of its characteristic equation.

Case 1 : **Two distinct real roots**

If the characteristic equation has two distinct real roots r_1 and r_2, then the general solution is

$$y(x) = c_1x^{r_1} + c_2x^{r_2}$$

Case 2 : **A double real root**

If the characteristic equation has a double real root r, then the general solution is

$$y(x) = c_1x^r + c_2x^r \ln x$$

Case 3 : **Two complex conjugate roots**

If the characteristic equation has two complex conjugate roots $\alpha + i\beta$ and $\alpha - i\beta$, then the general solution is

$$y(x) = c_1x^\alpha \cos(\beta \ln x) + c_2x^\alpha \sin(\beta \ln x)$$