Numerical Analysis PhD Qualifying Exam Syllabus

References:
Numerical Analysis by Timothy Sauer, Addison Wesley.
Numerical Mathematics and Computing (5th edition) by Cheney and Kincaid, Brooks/Cole.
Introduction to Scientific Computing by Charles F. van Loan, Prentice Hall.
`http://www.emba.uvm.edu/~lakobati/math337_MAIN.html`

237 Topics:

- Number Representation and Errors (Floating point, machine epsilon, sources of error, loss of significance, Taylor series, order of convergence)
- Locating Roots of Equations (Bisection, Newton, Fixed-Point Iteration, Secant)
- Interpolation (Lagrange, Newton, Cubic Splines)
- Differentiation and Integration (Finite Difference, Newton-Cotes, Adaptive Quadrature, Gaussian Quadrature)
- Linear Systems (Gaussian Elimination, LU, norms, SVD, Iterative Methods [Jacobi, Gauss-Seidel, SOR])
- Least Squares (Normal Equations, QR-Decomposition)
- Solution of Differential Equations (Explicit and Implicit methods, Runge-Kutta, Multi-Step methods)

337 Topics:

- Derivation of the local truncation error of a finite-difference method; The general relation between the global and local truncation errors for a first-order equation.
- Stability analysis of finite-difference methods.
- Methods for higher-order ODEs and systems of ODEs; The concept of a symplectic method.
- Basic existence and uniqueness properties of solution of a linear boundary-value problem (BVP).
- The simple shooting method for linear and nonlinear BVPs.
- Finite-difference methods for linear BVPs with Dirichlet and Neumann boundary conditions; Picard and Newton–Raphson iterative methods for nonlinear BVPs with Dirichlet boundary conditions.
- The θ-family of methods for the Heat equation with Dirichlet boundary conditions, and especially the Crank–Nicolson method.
- Von Neumann stability analysis for linear partial differential equations (PDEs).
- Crank–Nicolson method for linear parabolic PDEs with non-constant coefficients.
- Peaceman–Rachford method for the Heat equations in 2D with Dirichlet boundary conditions.
- Up to now, I have been unable to systematically cover an introduction to the method of characteristics for hyperbolic PDEs (and I don't think it is feasible to cover finite-difference methods for them in any reasonable depth). If, however, any method for hyperbolic PDEs is covered, it can/should be included in this syllabus.