Differential Equations PhD Qualifying Exam Syllabus

References:
Elementary Differential Equations by Boyce and DiPrima, or similar.
Nonlinear Dynamics and Chaos by Steve Strogatz.
Partial Differential Equations–Theory and Technique by Carrier and Pearson.
Applied Partial Differential Equations by Richard Haberman.

Instructions and passing criteria:
The exam will have two parts: one based on the math 230/330 material, and one based on the math 339 material. You will have four hours to complete the exam.
PhD passing criteria:
Two (2) problems from each part, for the total of four (4) problems, must be completed, and one (1) problem must be attempted. A problem is considered completed/attempted is a numeric score for it is 85%/60% or greater.
More specifically, to have attempted a problem, you must correctly outline the main idea of the solution and begin the calculation, but need not finish it.
MS passing criteria:
MS candidates will be evaluated on the 230/330 material only. They must complete two (2) problems and attempt (see above) one (1) additional problem.

230/330 Topics:
- Solution of linear and separable first-order equations. Particular and general solutions of linear equations. Solution to an initial-value problem.
- Second-order linear equations. Linear independence of solutions. Basis for the solution space. Wronskian. Statement and proof of Abel's theorem.
- Systems of linear differential equations. Conversion of a higher-order equation to a system of first-order equations in vector form. Solution of constant-coefficient first-order systems using eigenvalues and eigenvectors.
- Inhomogeneous linear equations, scalar and vector. Method of variation of parameters.
- Using Laplace transform to solve initial-value problems for ordinary differential equations.
- Phase plane analysis of dynamical systems. Classification of fixed (critical) points.
- Bifurcation theory. One and two dimensional bifurcations. Hopf bifurcations.
- Multiple-scale perturbation theory. Applications to weakly nonlinear oscillators.

339 Topics:
- Elliptic, parabolic, and hyperbolic partial differential equations.
- The classical equations: heat equation, wave equation, Laplace equation.
- Separation of variables. Fourier series expansions. Eigenfunction expansion.
- Strum-Liouville theory.
- D'Alembert's solution of the second-order wave equation.
- Method of characteristics. Solution of linear first-order hyperbolic equations with non-constant coefficients. Prediction of time and location of a shock in a nonlinear first-order hyperbolic equation.
- Fourier transform. Fourier and Laplace transforms of a product and of a convolution.