Numerical Analysis PhD and MS Qualifying Exam Syllabus

References:
Numerical Analysis by Timothy Sauer
Numerical Mathematics and Computing by Ward Cheney and David Kincaid
https://tlakoba.w3.uvm.edu/math337 *Lecture Notes for Math 337* by T.I. Lakoba

Instructions and passing criteria:

Seven (7) problems will be assigned, of which problems 1–4 will be based on the 237 material and problems 5–7 will be based on the 337 material. You will have three hours to complete the exam.

PhD passing criteria:
Four (4) problems must be completed, and one (1) problem must be attempted. At least two (2) problems from ## 1–4 (group 1) *and* at least two (2) problems from ## 4–7 (group 2) must be completed. Note that Problem 4 can count towards either group, but not both.
To have attempted a problem, you must correctly outline the main idea of the solution and begin the calculation, but need not finish it.

MS passing criteria:
MS candidates will be evaluated on the 237 material *only* (i.e., on Problems 1–4). They must complete two (2) problems and attempt (see above) one (1) additional problem.

237 Topics:

- Number Representation and Errors (Floating point, machine epsilon, sources of error, loss of significance, Taylor series, order of convergence)
- Locating Roots of Equations (Bisection, Newton, Fixed-Point Iteration, Secant)
- Interpolation (Lagrange, Newton, Cubic Splines)
- Differentiation and Integration (Finite Difference, Newton–Cotes, Adaptive Quadrature, Gaussian Quadrature)
- Linear Systems (Gaussian Elimination, LU, norms, SVD, Iterative Methods: Jacobi, Gauss–Seidel, and SOR)
- Least Squares (Normal Equations, QR Decomposition)
- Solution of Differential Equations (Explicit and Implicit methods, Runge–Kutta methods)

337 Topics:

- Derivation of the local truncation error of a finite-difference method; The general relation between the global and local truncation errors for a first-order equation.
- Stability analysis of finite-difference methods.
- Methods for higher-order ODEs and systems of ODEs; The concept of a symplectic method.
- Basic existence and uniqueness properties of solution of a linear boundary-value problem (BVP).
- The simple shooting method for linear and nonlinear BVPs.
- Finite-difference methods for linear BVPs with Dirichlet and Neumann boundary conditions; Picard and Newton–Raphson iterative methods for nonlinear BVPs with Dirichlet boundary conditions.
- The θ-family of methods for the Heat equation with Dirichlet boundary conditions, and especially the Crank–Nicolson method.
- Von Neumann stability analysis for linear partial differential equations (PDEs).
- Crank–Nicolson method for linear parabolic PDEs with non-constant coefficients.
- Peaceman–Rachford method for the Heat equations in 2D with Dirichlet boundary conditions.
- Method of Characteristic for a unidirectional wave equation (possibly with a forcing term).