Numerical Analysis PhD and MS Qualifying Exam Syllabus

References:
Numerical Analysis by Timothy Sauer
Numerical Mathematics and Computing by Ward Cheney and David Kincaid
https://tlakoba.w3.uvm.edu/math337 *Lecture Notes for Math 337* by T.I. Lakoba
(337 was the number of Math 6337 prior to Fall 2023.)

Instructions and passing criteria:
Seven (7) problems will be assigned, of which problems 1–4 will be based on Math 3337/5337 material and problems 5–7 will be based on the 6337 material. You will have 4 hours to complete the exam.
<u>*PhD passing criteria*</u>:
Four (4) problems must be completed, and one (1) problem must be attempted. At least two (2) problems from ## 1–4 (group 1) and at least two (2) problems from ## 4–7 (group 2) must be completed. Note that Problem 4 can count towards either group, but not both. To have attempted a problem, you must correctly outline the main idea of the solution and begin the calculation, but need not finish it.
<u>*MS passing criteria*</u>:
MS candidates will be evaluated on the 3337/5337 material only (i.e., on Problems 1–4). They must complete two (2) problems and attempt (see above) one (1) additional problem.

3337/537 Topics:
- Number Representation and Errors (Floating point, machine epsilon, sources of error, loss of significance, Taylor series, order of convergence)
- Locating Roots of Equations (Bisection, Newton, Fixed-Point Iteration, Secant methods) and Systems of Equations (Newton)
- Interpolation (Lagrange, Newton, Cubic Splines)
- Differentiation and Integration (Finite Difference, Newton–Cotes, Adaptive Quadrature, Gaussian Quadrature)
- Linear Systems (Gaussian Elimination, LU, norms, SVD, Iterative Methods: Jacobi, Gauss–Seidel, and SOR)
- Least Squares (Normal Equations, QR Decomposition)
- Solution of Differential Equations (Explicit and Implicit methods, Runge–Kutta methods)

6337 Topics:
- Derivation of the local truncation error of a finite-difference method; The general relation between the global and local truncation errors for a first-order equation.
- Stability analysis of finite-difference methods for ODEs.
- Methods for higher-order ODEs and systems of ODEs; The concept of a symplectic method.
- Basic existence and uniqueness properties of solution of a linear boundary-value problem (BVP).
- The simple shooting method for linear and nonlinear BVPs.
- Finite-difference methods for linear BVPs with Dirichlet and Neumann boundary conditions, maintaining the second-order accuracy at the boundary.
- Picard and Newton–Raphson iterative methods for nonlinear BVPs with Dirichlet boundary conditions.
- The θ-family of methods for the Heat equation with Dirichlet boundary conditions, and especially the Crank–Nicolson method.
- Von Neumann stability analysis for linear partial differential equations (PDEs).
- Crank–Nicolson method for linear parabolic PDEs with non-constant coefficients.
- Peaceman–Rachford method for the Heat equations in 2D with Dirichlet boundary conditions.
- Method of Characteristic for a unidirectional wave equation (possibly with a forcing term).