COMBINED REAL ANALYSIS AND COMPLEX ANALYSIS
PHD QUALIFYING EXAM SYLLABUS

The student should know the statement and how to apply every theorem named. He/She should be able to sketch proofs of the theorems marked with asterisks.

Real analysis.

Properties of $\mathbf{R}$ *and* $\mathbf{R}^n$.
Order axioms; least upper bound (sup and inf); convergence of sequences; Cauchy sequences; completeness; inner product and distance in $\mathbf{R}^n$; Cauchy-Schwarz inequality and triangle inequality; lim sup and lim inf; series and convergence tests.

Topology of $\mathbf{R}^n$.
Interior and boundary points; accumulation points; neighborhoods; open and closed sets; connected and path-connected sets; compact sets; Bolzano-Weierstrass and Heine-Borel theorems; metric spaces and topological spaces.

Continuity.
Limit of a function at a point; limit, open set, and sequential definitions of continuity; continuity of polynomials, rational functions, trig functions; continuous images of compact and connected sets; *Intermediate Value Theorem; uniform continuity and its relation to compact sets.

Differentiation in $\mathbf{R}^n$.
Partial derivatives; Jacobian; chain rule; gradient; directional derivatives; Taylor's Theorem with remainder in several variables; maxima and minima; Implicit and Inverse Function Theorems; Lagrange multipliers.

Riemann integration.
Partitions; upper and lower sums; definition of the integral and Riemann's criterion; integrability of continuous functions; *Fundamental Theorem of Calculus; improper integrals; multiple integrals; change of variables.

Sequences and series of functions.
Pointwise and uniform convergence; continuity, differentiability, and integrability of limits of functions; real power series; radii of convergence; convergence tests; Weierstrass M-test; differentiation and integration of series; spaces of continuous functions.

Measure theory.
Statements and applications of: Fatou's Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem.

Functional Analysis.
Normed linear spaces; Hilbert spaces; L^p spaces; Hölder and Minkowski inequalities; completeness; duals of L^p.

Complex analysis.

Basics.
Complex arithmetic; absolute value; polar decomposition; definitions of analytic; Cauchy-Riemann equations and relation to differentiation on $\mathbf{R}^2$; definition and basic properties of trig and exponential functions; Weierstrass M-test.

Complex integration.
Path and line integrals; *Goursat's Theorem; Cauchy's Theorem on disks and simply connected domains; Cauchy integral formula; power series expansions; zeroes and poles of meromorphic functions; isolated zeroes and singularities; Open Mapping Theorem; Maximum Modulus Theorem; *Morera's Theorem; *Liouville's Theorem; Laurent expansions; analytic logarithms.

Singularities.
Removable, pole, and essential singularities; *Casorati-Weierstrass Theorem; residues; computing residues for simple and multiple poles; using residues to find integrals; meromorphic functions; analytic continuation; Rouche's Theorem.

Conformal mappings.
Basic properties of conformal mappings; *Schwarz Lemma; analytic automorphisms of the unit disk; analytic mappings of the upper half-plane; fractional linear transformations; conformal mappings of strips, sectors, and slit planes onto the unit disk.

Harmonic functions.
Definition; definition and existence of harmonic conjugates; mean value property; maximum and minimum principles; Dirichlet Problem on the unit disk.