The student should know the statement and how to apply every theorem named. He/She should be able to sketch proofs of the theorems marked with asterisks.

**Real analysis.**

*Properties of \( \mathbb{R} \) and \( \mathbb{R}^n \).*

Order axioms; least upper bound (sup and inf); convergence of sequences; Cauchy sequences; completeness; inner product and distance in \( \mathbb{R}^n \); Cauchy-Schwarz inequality and triangle inequality; \( \lim \sup \) and \( \lim \inf \); series and convergence tests.

*Topology of \( \mathbb{R}^n \).*

Interior and boundary points; accumulation points; neighborhoods; open and closed sets; connected and path-connected sets; compact sets; Bolzano-Weierstrass and Heine-Borel theorems; metric spaces and topological spaces.

*Continuity.*

Limit of a function at a point; limit, open set, and sequential definitions of continuity; continuity of polynomials, rational functions, trig functions; continuous images of compact and connected sets; *Intermediate Value Theorem; uniform continuity and its relation to compact sets.*

*Differentiation in \( \mathbb{R}^n \).*

Partial derivatives; Jacobian; chain rule; gradient; directional derivatives; Taylor’s Theorem with remainder in several variables; maxima and minima; Implicit and Inverse Function Theorems; Lagrange multipliers.

*Riemann integration.*

Partitions; upper and lower sums; definition of the integral and Riemann’s criterion; integrability of continuous functions; *Fundamental Theorem of Calculus; improper integrals; multiple integrals; change of variables.*

*Sequences and series of functions.*

Pointwise and uniform convergence; continuity, differentiability, and integrability of limits of functions; real power series; radii of convergence; convergence tests; Weierstrass \( M \)-test; differentiation and integration of series; spaces of continuous functions.

*Measure theory.*

Statements and applications of: Fatou’s Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem.

*Functional Analysis.*

Normed linear spaces; Hilbert spaces; \( L^p \) spaces; Hölder and Minkowski inequalities; completeness; duals of \( L^p \).
Complex analysis.

*Basics.*
Complex arithmetic; absolute value; polar decomposition; definitions of analytic; Cauchy-Riemann equations and relation to differentiation on $\mathbb{R}^2$; definition and basic properties of trig and exponential functions; Weierstrass $M$-test.

*Complex integration.*
Path and line integrals; *Goursat’s Theorem; Cauchy’s Theorem on disks and simply connected domains; Cauchy integral formula; power series expansions; zeroes and poles of meromorphic functions; isolated zeroes and singularities; Open Mapping Theorem; Maximum Modulus Theorem; *Morera’s Theorem; *Liouville’s Theorem; Laurent expansions; analytic logarithms.

*Singularities.*
Removable, pole, and essential singularities; *Casorati-Weierstrass Theorem; residues; computing residues for simple and multiple poles; using residues to find integrals; meromorphic functions; analytic continuation; Rouche’s Theorem.

*Conformal mappings.*
Basic properties of conformal mappings; *Schwarz Lemma; analytic automorphisms of the unit disk; analytic mappings of the upper half-plane; fractional linear transformations; conformal mappings of strips, sectors, and slit planes onto the unit disk.

*Harmonic functions.*
Definition; definition and existence of harmonic conjugates; mean value property; maximum and minimum principles; Dirichlet Problem on the unit disk.