Numerical Analysis: MATH 337

Qualifying Examination

January 10, 2018

1. The Dahlquist method

$$Y_{n+1} - 2Y_n + Y_{n-1} \;=\; \frac{h^2}{4}\,(f_{n+1} + 2f_n + f_{n-1}), \qquad \text{where} \quad f_n \equiv f(x_n, Y_n), \text{ etc.} \tag{1}$$

can be used to solve the second-order initial-value problem

$$y'' = f(x, y), \qquad y(x_0) = y_0, \quad y'(x_0) = y'_0. \tag{2}$$

Show that method (1) has the global error of order 2 when applied to (2).

2. Consider the boundary-value problem

$$u''(x) + 13u'(x) + 17u(x) = R(x), \qquad u'(0) = \alpha, \qquad u(1) = \beta\,,$$

where $R(x)$ is an arbitrary given function and α and β are arbitrary given constants. Note the Neumann boundary condition at the left end point.

Write down the equations of a method that would approximate the solution of this problem with accuracy $O(h^2)$. You may choose *any* of the methods considered in the course MATH 337. Provide all necessary explanations.

3. Consider a unidirectional wave equation

$$u_t = c\,u_x, \qquad -\infty < x < \infty \tag{1}$$

where c =const.

Use the von Neumann analysis to determine under what condition on the ratio

$$\mu \;=\; \frac{c\kappa}{h}$$

the scheme

$$\frac{U_m^{n+1} - U_m^n}{\kappa} \;=\; c\,\frac{U_{m+1}^n - U_m^n}{h}\,, \tag{2}$$

approximating (1), is stable.