UVM - Algebra Qualifying Exam Topics

February 12, 2019

The material here can be found in Dummit and Foote.

Group Theory basic properties of groups; dihedral groups; cycle decomposition and symmetric groups; subgroups, cosets, and Lagrange's Theorem; cyclic groups and subgroups; subgroup lattice; quotient groups and normal subgroups; Isomorphism Theorems; Hölder Program and simple groups; transpositions, alternating groups; group actions, Cayley's Theorem, Class Equation; Sylow's Theorem and applications; simplicity of A_n; direct products, the Fundamental Theorem of Finitely Generated Abelian Groups and applications; commutators, the derived series, solvable groups; semidirect products.

Ring Theory basic properties of rings and subrings; homomorphisms, isomorphisms, quotient rings and the Isomorphism Theorems; ideals, prime ideals, maximal ideals, integral domains, construction of finite fields; rings of fractions; Chinese Remainder Theorem; Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains, quadratic integer rings; polynomial rings, factorization in one variable, Gauss' Lemma, irreducibility criteria, Eisenstein's Criterion; Noetherian rings.

Field Theory field extensions, degrees, algebraic and transcendental extensions, minimal polynomials of algebraic elements; straightedge and compass constructions; splitting fields and algebraic closures; existence and uniqueness of finite fields; separable extensions; cyclotomic polynomials.

Galois Theory field automorphisms, fixed fields and the Fundamental Theorem of Galois Theory, application in examples of small degree; Galois theory of finite fields; abelian extensions of $\mathbf{Q}$; Primitive Element Theorem; Fundamental Theorem of Algebra; symmetric functions and the Fundamental Theorem of Symmetric Functions; discriminants; roots of polynomials of degree ≤ 4; solvable and radical extensions; insolvability of the quintic.

Module Theory basic properties of modules and submodules; quotient modules and the Isomorphism Theorems; generation of modules, direct sums, and free modules; modules over Principal Ideal Domains, rank, and the Fundamental Theorem of Finitely Generated Modules over a P.I.D.

Linear Algebra vector spaces, spanning sets, independence, bases; subspaces, quotient spaces; linear transformations and their matrix representation; Gauss–Jordan elimination, computation of the image, rank, nullity and kernel of a linear transformation; change of basis, similarity; determinants; eigenvalues and eigenvectors; characteristic and minimal polynomials; Rational and Jordan Canonical Forms; dual spaces and double dual.