

Small-world networks

Last updated: 2020/09/12, 12:45:25 EDT

Principles of Complex Systems, Vol. 1 | @pocsvox
CSYS/MATH 300, Fall, 2020

PoCS, Vol. 1
Small-world
networks
1 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



Licensed under the *Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License*.

These slides are brought to you by:

Sealie & Lambie
Productions



PoCS, Vol. 1
Small-world
networks
2 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

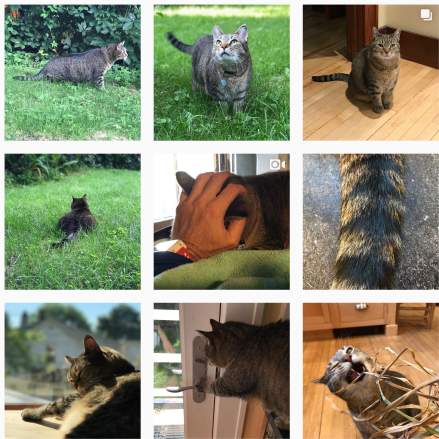
Nutshell



References



These slides are also brought to you by:

Special Guest Executive Producer



 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 

PoCS, Vol. 1
Small-world
networks
3 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Outline

PoCS, Vol. 1
Small-world
networks
4 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Small-world networks

Experiments

Theory

Generalized affiliation networks

Nutshell

References



Outline

PoCS, Vol. 1
Small-world
networks
6 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Small-world networks

Experiments

Theory

Generalized affiliation networks

Nutshell

References



People thinking about people:

How are social networks structured?

- 🧩 How do we define and measure connections?
- 🧩 Methods/issues of self-report and remote sensing.

PoCS, Vol. 1
Small-world
networks
7 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



People thinking about people:

How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

What about the dynamics of social networks?

- How do social networks/movements begin & evolve?
- How does collective problem solving work?
- How does information move through social networks?
- Which rules give the best 'game of society?'



People thinking about people:

How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

What about the dynamics of social networks?


- How do social networks/movements begin & evolve?
- How does collective problem solving work?
- How does information move through social networks?
- Which rules give the best 'game of society'?

Sociotechnical phenomena and algorithms:

- What can people and computers do together? (google)
- Use **Play + Crunch** to solve problems. Which problems?





A small slice of the pie:

 **Q.** Can people pass messages between distant individuals using only their existing social connections?



A small slice of the pie:

 **Q.** Can people pass messages between distant individuals using only their existing social connections?

 **A.** Apparently yes ...



Milgram's social search experiment (1960s)

PoCS, Vol. 1
Small-world
networks
9 of 70

Small-world
networks

Experiments

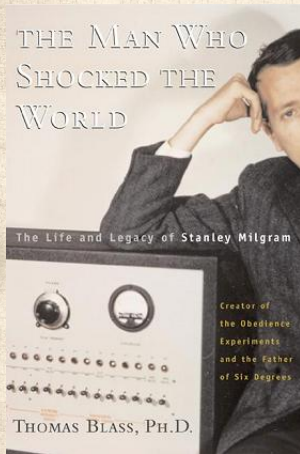
Theory

Generalized affiliation

networks

Nutshell

References



<http://www.stanleymilgram.com>



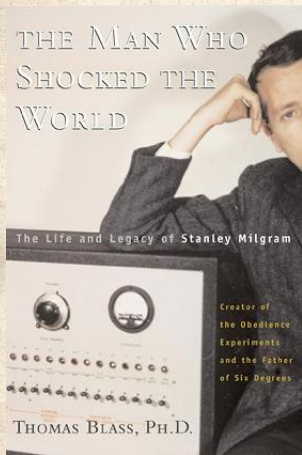
Target person =
Boston stockbroker.



296 senders from Boston
and Omaha.



Milgram's social search experiment (1960s)



<http://www.stanleymilgram.com>

- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- chain length ≈ 6.5 .

PoCS, Vol. 1
Small-world networks
9 of 70

Small-world networks

Experiments

Theory

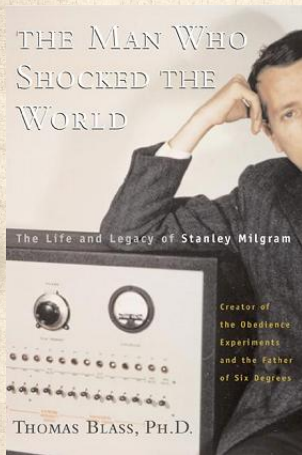
Generalized affiliation networks

Nutshell

References



Milgram's social search experiment (1960s)



<http://www.stanleymilgram.com>



Target person =
Boston stockbroker.



296 senders from Boston
and Omaha.



20% of senders reached
target.



chain length ≈ 6.5 .

Popular terms:



The Small World
Phenomenon;



“Six Degrees of Separation.”

PoCS, Vol. 1
Small-world
networks
9 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



From Frigyes Karinthy's "Chain-links" in "Everything is Different", 1929:

'A fascinating game grew out of this discussion. One of us suggested performing the following experiment to prove that the population of the Earth is closer together now than they have ever been before. We should select any person from the 1.5 billion inhabitants of the Earth—anyone, anywhere at all. He bet us that, using no more than five individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the network of personal acquaintances. For example, "Look, you know Mr. X.Y., please ask him to contact his friend Mr. Q.Z., whom he knows, and so forth."



Six Degrees of Kevin Bacon:

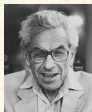


It's a game: "Kevin Bacon is the Center of the Universe"



The Oracle of Bacon

Six Degrees of Paul Erdős:



Academic papers.



Erdős Number



Erdős Number Project



Six Degrees of Kevin Bacon:

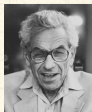


It's a game: "Kevin Bacon is the Center of the Universe"



The Oracle of Bacon

Six Degrees of Paul Erdős:




Academic papers.



Erdős Number





Erdős Number Project



 So naturally we must have the Erdős-Bacon Number.



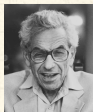
Six Degrees of Kevin Bacon:






 It's a game : "Kevin Bacon is the Center of the Universe"

 The Oracle of Bacon 



Six Degrees of Paul Erdős:




 Academic papers.

 Erdős Number 

 Erdős Number Project 



 So naturally we must have the Erdős-Bacon Number 



 One Story Lab alum has $EB\# < \infty$.



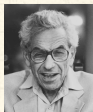
Six Degrees of Kevin Bacon:






 It's a game : "Kevin Bacon is the Center of the Universe"


 The Oracle of Bacon 



Six Degrees of Paul Erdős:





 Academic papers.

 Erdős Number 

 Erdős Number Project 

 So naturally we must have the Erdős-Bacon Number 

 One Story Lab alum has $EB\# < \infty$.

 Natalie Hershlag's (Portman's) $EB\# = 5 + 2 = 7$.



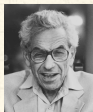
Six Degrees of Kevin Bacon:



It's a game: "Kevin Bacon is the Center of the Universe"

The Oracle of Bacon

Six Degrees of Paul Erdős:



Academic papers.

Erdős Number

Erdős Number Project

So naturally we must have the Erdős-Bacon Number.

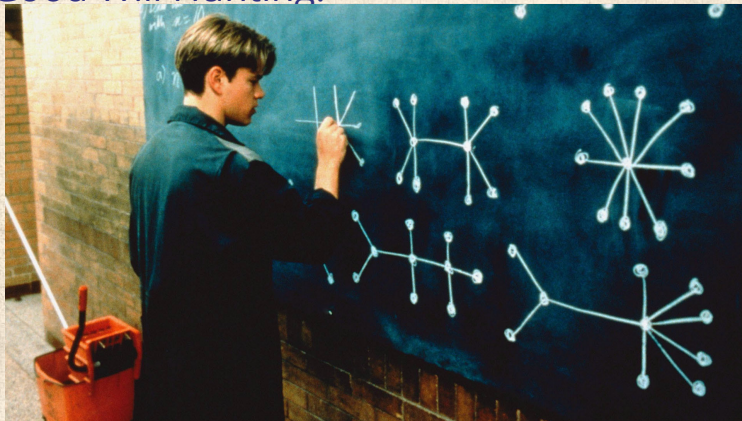
One Story Lab alum has $EB\# < \infty$.

Natalie Hershlag's (Portman's) $EB\# = 5 + 2 = 7$.

The EBS# is also a thing: erdosbaconsabbath.com



Good Will Hunting:



PoCS, Vol. 1
Small-world
networks
12 of 70

Small-world
networks

Experiments



Theory



Generalized affiliation
networks

Nutshell

References



 Boardwork by Dan Kleitman ,
 $EB\# = 1 + 2 = 3$.

 See Kleitman's sidebar in
Mark Saul's Movie Review 
(Notices of the AMS, Vol. 45,
1998.)



You may already be a winner in NSA's "three-degrees" surveillance sweepstakes!

NSA's probes could cover hundreds of millions of Americans. Thanks, Kevin Bacon.

by Sean Gallagher - July 18 2013, 4:00pm EDT

BIG DATA 109



Aurich Lawson

PoCS, Vol. 1
Small-world
networks
13 of 70

Small-world
networks

Experiments



Theory

Generalized affiliation
networks

Nutshell

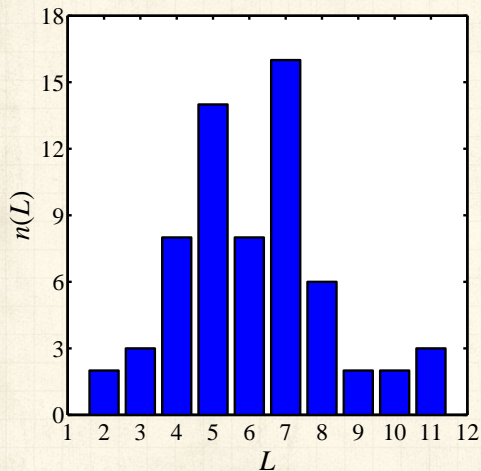
References



 Many people  are within three degrees from a random person ...

The problem

Lengths of successful chains:



From Travers and
Milgram (1969) in
Sociometry:^[12]
"An Experimental
Study of the Small
World Problem."



The problem

PoCS, Vol. 1
Small-world
networks
15 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Two features characterize a social 'Small World':



The problem

PoCS, Vol. 1
Small-world
networks
15 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Two features characterize a social 'Small World':

1. Short paths exist,
and



The problem

PoCS, Vol. 1
Small-world
networks
15 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Two features characterize a social 'Small World':

1. Short paths exist,
and
2. People are good at finding them.



The problem

PoCS, Vol. 1
Small-world
networks
15 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Two features characterize a social 'Small World':

1. Short paths exist, (= Geometric piece)
and
2. People are good at finding them.



The problem

PoCS, Vol. 1
Small-world
networks
15 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Two features characterize a social 'Small World':

1. Short paths exist, (= Geometric piece)
and
2. People are good at finding them. (= Algorithmic piece)



Social Search

Milgram's small world experiment with email:

Events and News
Duncan J. Watts's new book is out now!

Project Information
In the Press
Description
Procedures
Security and Privacy
Articles/References
Results

Research Team
Duncan J. Watts
Peter Dodds
Roby Muhamad

Web Development
Peter Hauxel

Vijay (Delhi, India) worked at an engineering firm with a friend.

Sneer (Kolkata, India) whose daughter is a friend of yours.

Pema (Berkeley, USA) goes to school in California and plays soccer with a friend.

Alice (New York, USA) whose friend is a friend of yours.

Christina (Bronx, USA) whose best friend from high school is a friend of yours.

William (New York, NY) is studying medicine with a friend of yours.

home
my small world
chat
FAQ
related links

login
sign up

The **SMALL WORLD** project is an online experiment to test the idea that any two people in the world can be connected via "six degrees of separation".

Your objective is to get a message to a "target person", somewhere in the world, by forwarding the message to a friend of yours—someone who is "closer" to the target than you are. (If you happen know the target, you can of course send it to them)

If we have asked you to participate (you would have received a message from a friend of yours), you should **continue** the chain.

If you are just visiting us, sign up to start a new chain.

COLUMBIA UNIVERSITY

PoCS, Vol. 1
Small-world
networks
16 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation

networks

Nutshell

References




"An Experimental Study of Search in Global
Social Networks" 

Dodds, Muhamad, and Watts,
Science, **301**, 827–829, 2003. ^[6]



Social search—the Columbia experiment

 60,000+ participants in 166 countries

PoCS, Vol. 1
Small-world
networks
17 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation
networks


Nutshell

References



Social search—the Columbia experiment

 60,000+ participants in 166 countries

 18 targets in 13 countries including

PoCS, Vol. 1
Small-world
networks
17 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Social search—the Columbia experiment

- 60,000+ participants in 166 countries
- 18 targets in 13 countries including
 - a professor at an Ivy League university,

PoCS, Vol. 1
Small-world
networks
17 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation
networks


Nutshell



References



Social search—the Columbia experiment


 60,000+ participants in 166 countries


 18 targets in 13 countries including




-  a professor at an Ivy League university,
-  an archival inspector in Estonia,



Social search—the Columbia experiment


 60,000+ participants in 166 countries


 18 targets in 13 countries including





-  a professor at an Ivy League university,
-  an archival inspector in Estonia,
-  a technology consultant in India,



Social search—the Columbia experiment


 60,000+ participants in 166 countries


 18 targets in 13 countries including






-  a professor at an Ivy League university,
-  an archival inspector in Estonia,
-  a technology consultant in India,
-  a policeman in Australia,



Social search—the Columbia experiment


 60,000+ participants in 166 countries


 18 targets in 13 countries including






-  a professor at an Ivy League university,
-  an archival inspector in Estonia,
-  a technology consultant in India,
-  a policeman in Australia,
and
-  a veterinarian in the Norwegian army.




Social search—the Columbia experiment

 60,000+ participants in 166 countries


 18 targets in 13 countries including


-  a professor at an Ivy League university,
-  an archival inspector in Estonia,
-  a technology consultant in India,
-  a policeman in Australia,
and
-  a veterinarian in the Norwegian army.






 24,000+ chains




Social search—the Columbia experiment

 60,000+ participants in 166 countries

 18 targets in 13 countries including

-  a professor at an Ivy League university,
-  an archival inspector in Estonia,
-  a technology consultant in India,
-  a policeman in Australia,
and
-  a veterinarian in the Norwegian army.

 24,000+ chains

We were lucky and contagious (more later):

[“Using E-Mail to Count Connections”](#) , Sarah Milstein,
New York Times, Circuits Section (December, 2001)



All targets:

PoCS, Vol. 1
Small-world
networks
18 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Table S1

Target	City	Country	Occupation	Gender	N	N_c (%)	r (ro)	<L>
1	Novosibirsk	Russia	PhD student	F	8234	20(0.24)	64 (76)	4.05
2	New York	USA	Writer	F	6044	31 (0.51)	65 (73)	3.61
3	Bandung	Indonesia	Unemployed	M	8151	0	66 (76)	n/a
4	New York	USA	Journalist	F	5690	44 (0.77)	60 (72)	3.9
5	Ithaca	USA	Professor	M	5855	168 (2.87)	54 (71)	3.84
6	Melbourne	Australia	Travel Consultant	F	5597	20 (0.36)	60 (71)	5.2
7	Bardufoss	Norway	Army veterinarian	M	4343	16 (0.37)	63 (76)	4.25
8	Perth	Australia	Police Officer	M	4485	4 (0.09)	64 (75)	4.5
9	Omaha	USA	Life Insurance Agent	F	4562	2 (0.04)	66 (79)	4.5
10	Welwyn Garden City	UK	Retired	M	6593	1 (0.02)	68 (74)	4
11	Paris	France	Librarian	F	4198	3 (0.07)	65 (75)	5
12	Tallinn	Estonia	Archival Inspector	M	4530	8 (0.18)	63(79)	4
13	Munich	Germany	Journalist	M	4350	32 (0.74)	62 (74)	4.66
14	Split	Croatia	Student	M	6629	0	63 (77)	n/a
15	Gurgaon	India	Technology Consultant	M	4510	12 (0.27)	67 (78)	3.67
16	Managua	Nicaragua	Computer analyst	M	6547	2 (0.03)	68 (78)	5.5
17	Katikati	New Zealand	Potter	M	4091	12 (0.3)	62 (74)	4.33
18	Elderton	USA	Lutheran Pastor	M	4438	9 (0.21)	68 (76)	4.33
Totals					98,847	384 (0.4)	63 (75)	4.05



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
19 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Milgram's participation rate was roughly 75%



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
19 of 70

Small-world
networks



Experiments

Theory

Generalized affiliation
networks

Nutshell

References

-  Milgram's participation rate was roughly 75%
-  Email version: Approximately 37% participation rate.



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
19 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

- ✉ Milgram's participation rate was roughly 75%
- ✉ Email version: Approximately 37% participation rate.
- ✉ Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
19 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

- ✉ Milgram's participation rate was roughly 75%
- ✉ Email version: Approximately 37% participation rate.
- ✉ Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$

- ✉ \Rightarrow 384 completed chains (1.6% of all chains).



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
20 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Motivation/Incentives/Perception matter.



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
20 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

- ⊞ Motivation/Incentives/Perception matter.
- ⊞ If target *seems* reachable
⇒ participation more likely.



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
20 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

- ⊞ Motivation/Incentives/Perception matter.
- ⊞ If target *seems* reachable
⇒ participation more likely.
- ⊞ Small changes in attrition rates
⇒ large changes in completion rates



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
20 of 70

Small-world
networks


Experiments


Theory


Generalized affiliation
networks


Nutshell

References

 Motivation/Incentives/Perception matter.

 If target *seems* reachable
⇒ participation more likely.

 Small changes in attrition rates
⇒ large changes in completion rates


 e.g., ↘ 15% in attrition rate
⇒ ↗ 800% in completion rate

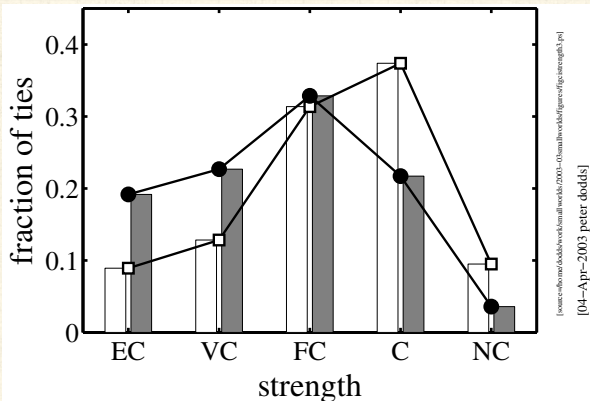


Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
21 of 70

Comparing successful to unsuccessful chains:

 Successful chains used relatively weaker ties:



Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Social search—the Columbia experiment

Successful chains disproportionately used:

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks


Nutshell

References



Social search—the Columbia experiment

Successful chains disproportionately used:

 Weak ties, Granovetter [7]

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell



References



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Successful chains disproportionately used:

-  Weak ties, Granovetter ^[7]
-  Professional ties (34% vs. 13%)

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments




Theory

Generalized affiliation
networks

Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter [7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments





Theory

Generalized affiliation
networks

Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter [7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments





Theory

Generalized affiliation
networks

Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter [7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)

...and disproportionately avoided



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments





Theory

Generalized affiliation
networks


Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter [7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)

...and disproportionately avoided

-  hubs (8% vs. 1%) (+ no evidence of funnels)



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments





Theory

Generalized affiliation
networks



Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter ^[7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)

...and disproportionately avoided

-  hubs (8% vs. 1%) (+ no evidence of funnels)
-  family/friendship ties (60% vs. 83%)



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
22 of 70

Small-world
networks

Experiments





Theory

Generalized affiliation
networks



Nutshell

References

Successful chains disproportionately used:

-  Weak ties, Granovetter ^[7]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)

...and disproportionately avoided


-  hubs (8% vs. 1%) (+ no evidence of funnels)
-  family/friendship ties (60% vs. 83%)

Geography → Work



Social search—the Columbia experiment

Senders of successful messages showed
little absolute dependency on

 age, gender

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks

Nutshell

References

Senders of successful messages showed
little absolute dependency on

 age, gender

 country of residence



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks


Nutshell

References

Senders of successful messages showed
little absolute dependency on

 age, gender

 country of residence

 income



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks


Nutshell


References

Senders of successful messages showed
little absolute dependency on

 age, gender

 country of residence

 income

 religion



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments






Theory

Generalized affiliation
networks

Nutshell

References

Senders of successful messages showed
little absolute dependency on

-  age, gender
-  country of residence
-  income
-  religion
-  relationship to recipient



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
23 of 70

Small-world
networks

Experiments






Theory

Generalized affiliation
networks

Nutshell

References

Senders of successful messages showed
little absolute dependency on

-  age, gender
-  country of residence
-  income
-  religion
-  relationship to recipient

Range of completion rates for subpopulations:

30% to 40%



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
24 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks

Nutshell

References

Mildly bad for continuing chain:

choosing recipients because “they have lots of friends”
or because they will “likely continue the chain.”



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
24 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks


Nutshell

References

Mildly bad for continuing chain:

choosing recipients because “they have lots of friends”
or because they will “likely continue the chain.”

Why:

 Specificity important



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
24 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks

Nutshell

References

Mildly bad for continuing chain:

choosing recipients because “they have lots of friends”
or because they will “likely continue the chain.”

Why:



Specificity important



Successful links used relevant information.
(e.g. connecting to someone who shares same
profession as target.)



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments


Theory

Generalized affiliation
networks

Nutshell

References

Basic results:

 $\langle L \rangle = 4.05$ for all completed chains



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks

Nutshell

References

Basic results:

 $\langle L \rangle = 4.05$ for all completed chains

 L_* = Estimated 'true' median chain length (zero attrition)



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks


Nutshell

References

Basic results:

 $\langle L \rangle = 4.05$ for all completed chains

 L_* = Estimated 'true' median chain length (zero attrition)

 Intra-country chains: $L_* = 5$



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation


networks


Nutshell


References

Basic results:

 $\langle L \rangle = 4.05$ for all completed chains

 L_* = Estimated 'true' median chain length (zero attrition)

 Intra-country chains: $L_* = 5$

 Inter-country chains: $L_* = 7$



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation


networks


Nutshell


References


Basic results:

 $\langle L \rangle = 4.05$ for all completed chains

 L_* = Estimated 'true' median chain length (zero attrition)

 Intra-country chains: $L_* = 5$

 Inter-country chains: $L_* = 7$

 All chains: $L_* = 7$



Social search—the Columbia experiment

PoCS, Vol. 1
Small-world
networks
25 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation


networks


Nutshell


References


Basic results:


 $\langle L \rangle = 4.05$ for all completed chains

 L_* = Estimated 'true' median chain length (zero attrition)

 Intra-country chains: $L_* = 5$

 Inter-country chains: $L_* = 7$

 All chains: $L_* = 7$

 Milgram: $L_* \simeq 9$



Usefulness:

Harnessing social search:

PoCS, Vol. 1
Small-world
networks
26 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks


Nutshell

References



Usefulness:

Harnessing social search:

 Can distributed social search be used for something big/good?

PoCS, Vol. 1
Small-world
networks
26 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Usefulness:




Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)



Usefulness:





Harnessing social search:

-  Can distributed social search be used for something big/good?
-  What about something evil? (Good idea to check.)
-  What about socio-inspired algorithms for information search? (More later.)



Usefulness:

Harnessing social search:

-  Can distributed social search be used for something big/good?
-  What about something evil? (Good idea to check.)
-  What about socio-inspired algorithms for information search? (More later.)
-  For real social search, we have an incentives problem.



Usefulness:

Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?



Usefulness:

Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?
- Fun, money, prestige, ...?



Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?
- Fun, money, prestige, ...?
- Must be 'non-gameable.'



Red balloons:

A Grand Challenge:

PoCS, Vol. 1
Small-world
networks
27 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks


Nutshell

References



Red balloons:

A Grand Challenge:

 1969: The Internet is born 
(the ARPANET —four nodes!).

PoCS, Vol. 1
Small-world
networks
27 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell



References




Red balloons:

A Grand Challenge:

 1969: The Internet is born 
(the ARPANET —four nodes!).




 Originally funded by DARPA who created a grand
Network Challenge  for the 40th anniversary.



*DARPA = Defense Advanced Research Projects Agency .

Red balloons:

A Grand Challenge:








-  1969: The Internet is born [↗](#)
(the ARPANET [↗](#)—four nodes!).
-  Originally funded by DARPA who created a grand Network Challenge [↗](#) for the 40th anniversary.
-  Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.



*DARPA = Defense Advanced Research Projects Agency [↗](#).

Red balloons:

A Grand Challenge:

-  1969: The Internet is born 
(the ARPANET —four nodes!).
-  Originally funded by DARPA who created a grand Network Challenge  for the 40th anniversary.
-  Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.
-  Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.



*DARPA = Defense Advanced Research Projects Agency .

Red balloons:

A Grand Challenge:

- 1969: The Internet is born (the ARPANET—four nodes!).
- Originally funded by DARPA who created a grand Network Challenge for the 40th anniversary.
- Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.
- Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.
- Challenge: Find the latitude and longitude of each balloon.



*DARPA = Defense Advanced Research Projects Agency.

Red balloons:

A Grand Challenge:

- 1969: The Internet is born (the ARPANET—four nodes!).
- Originally funded by DARPA who created a grand Network Challenge for the 40th anniversary.
- Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.
- Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.
- Challenge: Find the latitude and longitude of each balloon.
- Prize: **\$40,000**.

*DARPA = Defense Advanced Research Projects Agency.



Where the balloons were:



PoCS, Vol. 1
Small-world
networks
28 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks



Nutshell

References



Finding red balloons:

The winning team and strategy:

 MIT's Media Lab  won in less than 9 hours. [9]

PoCS, Vol. 1
Small-world
networks
29 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks




Nutshell

References



Finding red balloons:

The winning team and strategy:

-  MIT's Media Lab  won in less than 9 hours. [9]
-  Pickard et al. "Time-Critical Social Mobilization," [9]
Science Magazine, 2011.



Finding red balloons:






The winning team and strategy:

- 🧱 MIT's Media Lab [↗](#) won in less than 9 hours. [9]
- 🧱 Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
- 🧱 People were virally recruited online to help out.



Finding red balloons:







The winning team and strategy:

-  MIT's Media Lab  won in less than 9 hours. [9]
-  Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
-  People were virally recruited online to help out.
-  Idea: Want people to both (1) find the balloons, and (2) involve more people.



Finding red balloons:


The winning team and strategy:

-  MIT's Media Lab  won in less than 9 hours. [9]
-  Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
-  People were virally recruited online to help out.
-  Idea: Want people to both (1) find the balloons, and (2) involve more people.
-  Recursive incentive structure with exponentially decaying payout:



Finding red balloons:









The winning team and strategy:

- MIT's Media Lab  won in less than 9 hours. [9]
- Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
- People were virally recruited online to help out.
- Idea: Want people to both (1) find the balloons, and (2) involve more people.
- Recursive incentive structure with exponentially decaying payout:
 - \$2000 for correctly reporting the coordinates of a balloon.



Finding red balloons:

The winning team and strategy:

-  MIT's Media Lab  won in less than 9 hours. [9]
-  Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
-  People were virally recruited online to help out.
-  Idea: Want people to both (1) find the balloons, and (2) involve more people.
-  Recursive incentive structure with exponentially decaying payout:
 -  \$2000 for correctly reporting the coordinates of a balloon.
 -  \$1000 for recruiting a person who finds a balloon.



Finding red balloons:


The winning team and strategy:

- MIT's Media Lab  won in less than 9 hours. [9]
- Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
- People were virally recruited online to help out.
- Idea: Want people to both (1) find the balloons, and (2) involve more people.
- Recursive incentive structure with exponentially decaying payout:
 - \$2000 for correctly reporting the coordinates of a balloon.
 - \$1000 for recruiting a person who finds a balloon.
 - \$500 for recruiting a person who recruits the balloon finder, ...



Finding red balloons:



The winning team and strategy:


- MIT's Media Lab  won in less than 9 hours. [9]
- Pickard et al. "Time-Critical Social Mobilization," [9] Science Magazine, 2011.
- People were virally recruited online to help out.
- Idea: Want people to both (1) find the balloons, and (2) involve more people.
- Recursive incentive structure with exponentially decaying payout:
 - \$2000 for correctly reporting the coordinates of a balloon.
 - \$1000 for recruiting a person who finds a balloon.
 - \$500 for recruiting a person who recruits the balloon finder, ...
 - (Not a Ponzi scheme.)





Finding red balloons:


The winning team and strategy:





 [MIT's Media Lab](#)  won in less than 9 hours. [9]



 Pickard et al. "Time-Critical Social Mobilization," [9]
Science Magazine, 2011.

 People were virally recruited online to help out.

 Idea: Want people to both (1) find the balloons,
and (2) involve more people.

 Recursive incentive structure with exponentially
decaying payout:


-  \$2000 for correctly reporting the coordinates of a balloon.
-  \$1000 for recruiting a person who finds a balloon.
-  \$500 for recruiting a person who recruits the balloon finder, ...
-  (Not a Ponzi scheme.)

 True victory: [Colbert interviews Riley Crane](#) 



Finding balloons:

Clever scheme:

 Max payout = \$4000 per balloon.

PoCS, Vol. 1
Small-world
networks
30 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Finding balloons:

Clever scheme:



Max payout = \$4000 per balloon.



Individuals have clear incentives to both

1. **involve/source more people** (spread), and
2. **find balloons** (goal action).



Finding balloons:

Clever scheme:

- 🧱 Max payout = \$4000 per balloon.
- 🧱 Individuals have clear incentives to both
 1. **involve/source more people** (spread), and
 2. **find balloons** (goal action).
- 🧱 Gameable?



Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
 - involve/source more people (spread), and
 - find balloons (goal action).
- Gameable?
- Limit to how much money a set of bad actors can extract.



Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
 - involve/source more people (spread), and
 - find balloons (goal action).
- Gameable?
- Limit to how much money a set of bad actors can extract.

Extra notes:

- MIT's brand helped greatly.



Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
 - involve/source more people (spread), and
 - find balloons (goal action).
- Gameable?
- Limit to how much money a set of bad actors can extract.

Extra notes:

- MIT's brand helped greatly.
- MIT group first heard about the competition a few days before.



Finding balloons:

Clever scheme:

- 🧱 Max payout = \$4000 per balloon.
- 🧱 Individuals have clear incentives to both
 1. **involve/source more people** (spread), and
 2. **find balloons** (goal action).
- 🧱 Gameable?
- 🧱 Limit to how much money a set of bad actors can extract.

Extra notes:

- 🧱 MIT's brand helped greatly.
- 🧱 MIT group first heard about the competition a few days before. **Ouch.**




Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
 - involve/source more people (spread), and
 - find balloons (goal action).
- Gameable?
- Limit to how much money a set of bad actors can extract.

Extra notes:

- MIT's brand helped greatly.
- MIT group first heard about the competition a few days before. **Ouch.**
- A number of other teams did well .




Finding balloons:

Clever scheme:

- 🧱 Max payout = \$4000 per balloon.
- 🧱 Individuals have clear incentives to both
 1. **involve/source more people** (spread), and
 2. **find balloons** (goal action).
- 🧱 Gameable?
- 🧱 Limit to how much money a set of bad actors can extract.

Extra notes:

- 🧱 MIT's brand helped greatly.
- 🧱 MIT group first heard about the competition a few days before. **Ouch.**
- 🧱 A number of other teams did well .
- 🧱 Worthwhile looking at these competing strategies. ^[9]



Collective Detective:



Finding an errant panda

Once again, social media proved to be a powerful dragnet. Around 1:15 p.m., a Washingtonian posted a picture on Twitter of Rusty in a patch of weeds in the Adams Morgan district, not far from the 163-acre zoo, which was created in 1889 by an act of Congress. "Red panda in our neighborhood," [wrote Ashley Foughty](#), who identified herself as a singer, actress and traveler. "Please come save him!"

Another neighbor posted [a photograph](#) of two zoo workers, one in safari shorts standing on a rooftop, one holding a giant butterfly net. Soon the zoo announced: "Rusty the red panda has been recovered, crated & is headed safely back to the National Zoo!"



Collective Detective:




Finding an errant panda

Once again, social media proved to be a powerful dragnet. Around 1:15 p.m., a Washingtonian posted a picture on Twitter of Rusty in a patch of weeds in the Adams Morgan district, not far from the 163-acre zoo, which was created in 1889 by an act of Congress. "Red panda in our neighborhood," wrote Ashley Foughty, who identified herself as a singer, actress and traveler. "Please come save him!"

Another neighbor posted a photograph of two zoo workers, one in safari shorts standing on a rooftop, one holding a giant butterfly net. Soon the zoo announced: "Rusty the red panda has been recovered, crated & is headed safely back to the National Zoo!"



Nature News: "Crowdsourcing in manhunts can work: Despite mistakes over the Boston bombers, social media can help to find people quickly"  by Philip Ball (April 26, 2013)



Collective Detective:




Finding an errant panda


Once again, social media proved to be a powerful dragnet. Around 1:15 p.m., a Washingtonian posted a picture on Twitter of Rusty in a patch of weeds in the Adams Morgan district, not far from the 163-acre zoo, which was created in 1889 by an act of Congress. "Red panda in our neighborhood," [wrote Ashley Foughty](#), who identified herself as a singer, actress and traveler. "Please come save him!"

Another neighbor posted [a photograph](#) of two zoo workers, one in safari shorts standing on a rooftop, one holding a giant butterfly net. Soon the zoo announced: "Rusty the red panda has been recovered, crated & is headed safely back to the National Zoo!"



Nature News: "Crowdsourcing in manhunts can work: Despite mistakes over the Boston bombers, social media can help to find people quickly"  by Philip Ball (April 26, 2013)



Motherboard, Vice: One Degree of Separation in the Forever War  by Brian Castner (November 11, 2015)



Outline

PoCS, Vol. 1
Small-world
networks
32 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Small-world networks

Experiments

Theory

Generalized affiliation networks

Nutshell

References



The social world appears to be small ...why?

PoCS, Vol. 1
Small-world
networks
34 of 70

Small-world
networks

Experiments


Theory

Generalized affiliation
networks

Nutshell

References

Theory: how do we understand the small world property?

 Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

d_{AB} = distance between nodes A and B .



The social world appears to be small ...why?

PoCS, Vol. 1
Small-world
networks
34 of 70

Small-world
networks

Experiments


Theory

Generalized affiliation
networks

Nutshell

References


Theory: how do we understand the small world property?

 Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

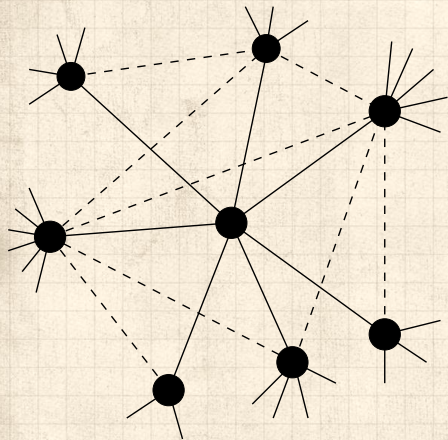
N = population size,

d_{AB} = distance between nodes A and B .

 **But: social networks aren't random ...**



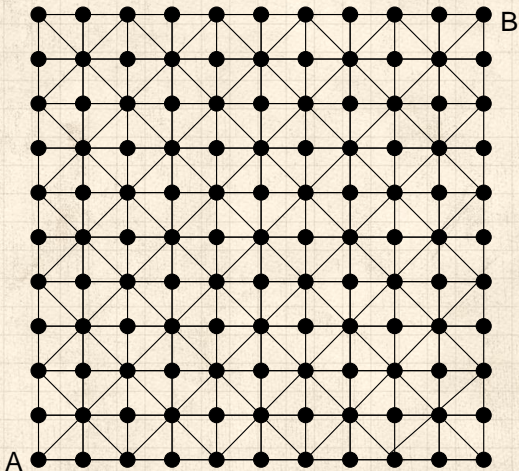
Simple socialness in a network:



Need “clustering”
(your friends are
likely to know each
other):



Non-randomness gives clustering:



$d_{AB} = 10 \rightarrow$ too many long paths.



Randomness + regularity

PoCS, Vol. 1
Small-world
networks
37 of 70

Small-world
networks

Experiments

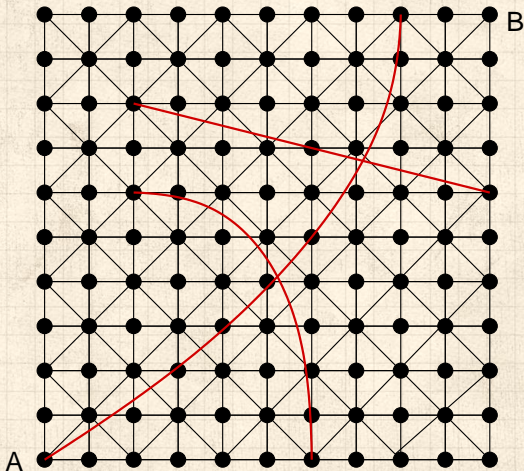
Theory

Generalized affiliation

networks

Nutshell

References



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall



Small-world networks

Introduced by Watts and Strogatz (Nature, 1998) [14]
"Collective dynamics of 'small-world' networks."

PoCS, Vol. 1
Small-world
networks
38 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998) [14]
"Collective dynamics of 'small-world' networks."

Small-world
networks


Experiments

Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

 neural network of C. elegans,

References



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998) [14]
"Collective dynamics of 'small-world' networks."

Small-world
networks



Experiments

Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

-  neural network of *C. elegans*,
-  semantic networks of languages,

References



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998) [14]
"Collective dynamics of 'small-world' networks."

Small-world
networks




Experiments

Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

-  neural network of C. elegans,
-  semantic networks of languages,
-  actor collaboration graph,

References



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998) [14]
"Collective dynamics of 'small-world' networks."

Small-world
networks





Experiments

Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

-  neural network of *C. elegans*,
-  semantic networks of languages,
-  actor collaboration graph,
-  food webs,

References



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998)^[14]
"Collective dynamics of 'small-world' networks."

Small-world
networks

Experiments






Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

References

-  neural network of C. elegans,
-  semantic networks of languages,
-  actor collaboration graph,
-  food webs,
-  social networks of comic book characters, ...



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998)^[14]
"Collective dynamics of 'small-world' networks."

Small-world
networks

Experiments

Theory






Generalized affiliation

networks


Nutshell

Small-world networks were found everywhere:

References

-  neural network of C. elegans,
-  semantic networks of languages,
-  actor collaboration graph,
-  food webs,
-  social networks of comic book characters, ...

Very weak requirements:

-  local regularity



Small-world networks

PoCS, Vol. 1
Small-world
networks
38 of 70

Introduced by Watts and Strogatz (Nature, 1998)^[14]
"Collective dynamics of 'small-world' networks."

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

Small-world networks were found everywhere:

References

- neural network of *C. elegans*,
- semantic networks of languages,
- actor collaboration graph,
- food webs,
- social networks of comic book characters, ...

Very weak requirements:

- local regularity + random short cuts



Papers should be apps:

PoCS, Vol. 1
Small-world networks
39 of 70

Small-world networks

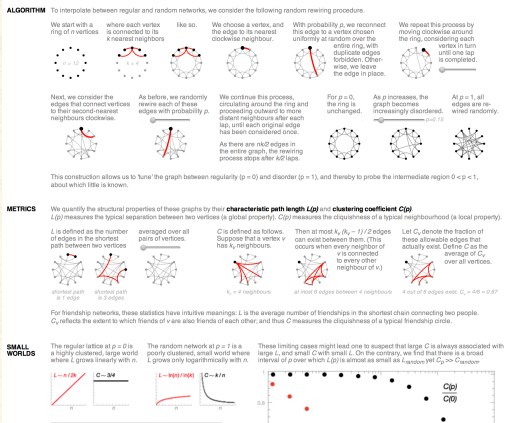
Experiments

Theory

Generalized affiliation networks

Nutshell

References



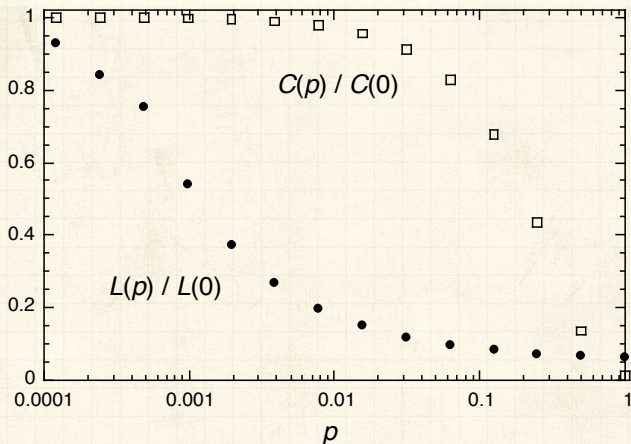
Bret Victor's Scientific Communication As Sequential Art





Interactive figures and tables = windows into large data sets (empirical or simulated).



The structural small-world property:



 $L(p)$ = average shortest path length as a function of p

 $C(p)$ = average clustering as a function of p



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
41 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

But are these short cuts findable?



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
41 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

But are these short cuts findable?

Nope. [8]



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
41 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

But are these short cuts findable?

Nope. [8]

Nodes **cannot** find each other quickly
with **any local search method**.



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
41 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

But are these short cuts findable?

Nope. [8]

Nodes **cannot** find each other quickly
with **any local search method**.

Need a more sophisticated model ...



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
42 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks

Nutshell

References



What can a local search method reasonably use?



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
42 of 70

Small-world
networks


Experiments


Theory

Generalized affiliation
networks

Nutshell

References

 What can a local search method reasonably use?

 How to find things without a map?



Previous work—finding short paths

- What can a local search method reasonably use?
- How to find things without a map?
- Need some measure of distance between friends and the target.



Previous work—finding short paths

- What can a local search method reasonably use?
- How to find things without a map?
- Need some measure of distance between friends and the target.

Some possible knowledge:

- Target's identity
- Friends' popularity
- Friends' identities
- Where message has been



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
43 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Jon Kleinberg (Nature, 2000) [8]
“Navigation in a small world.”



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
43 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Jon Kleinberg (Nature, 2000) [8]
“Navigation in a small world.”

Allowed to vary:

1. local search algorithm



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
43 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Jon Kleinberg (Nature, 2000) [8]
“Navigation in a small world.”

Allowed to vary:

1. local search algorithm
and
2. network structure.



Previous work—finding short paths

Kleinberg's Network:

PoCS, Vol. 1
Small-world
networks
44 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.

PoCS, Vol. 1
Small-world
networks
44 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .



Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .
3. Add m short cuts per node.



Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto x_{ij}^{-\alpha}.$$



Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto x_{ij}^{-\alpha}.$$



$\alpha = 0$: random connections.



α large: reinforce local connections.




$\alpha = d$: connections grow logarithmically in space.



Previous work—finding short paths

Theoretical optimal search:

 "Greedy" algorithm.

PoCS, Vol. 1
Small-world
networks
45 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks

Nutshell

References



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
45 of 70

Small-world
networks

Experiments


Theory


Generalized affiliation
networks

Nutshell

References

Theoretical optimal search:

 “Greedy” algorithm.

 Number of connections grow logarithmically
(slowly) in space: $\alpha = d$.



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
45 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation

networks

Nutshell

References

Theoretical optimal search:

- 🧱 “Greedy” algorithm.
- 🧱 Number of connections grow logarithmically (slowly) in space: $\alpha = d$.
- 🧱 Social golf.



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
45 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation


networks


Nutshell

References

Theoretical optimal search:

 "Greedy" algorithm.

 Number of connections grow logarithmically (slowly) in space: $\alpha = d$.

 Social golf.

Search time grows slowly with system size (like $\log^2 N$).



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
45 of 70

Small-world
networks

Experiments

Theory


Generalized affiliation


networks


Nutshell

References

Theoretical optimal search:

 “Greedy” algorithm.

 Number of connections grow logarithmically (slowly) in space: $\alpha = d$.

 Social golf.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.

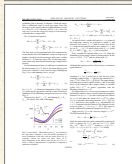



Advances for understanding Kleinberg's model:



"Kleinberg Navigation in Fractal Small World Networks" 


Roberson and ben-Avraham,
Phys. Rev. E, **74**, 017101, 2006. ^[10]



"Asymptotic behavior of the Kleinberg model" 

Carmi et al.,
Phys. Rev. Lett., **102**, 238702, 2009. ^[4]



"Extended navigability of small world networks: Exact results and new insights" 

Cartoza and De Los Rios,
Phys. Rev. Lett., **2009**, 238703, 2009. ^[5]



Previous work—finding short paths

PoCS, Vol. 1
Small-world
networks
47 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



If networks have **hubs** can also search well:
Adamic et al. (2001)^[1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).



Previous work—finding short paths

- 🧱 If networks have hubs can also search well:
Adamic et al. (2001)^[1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- 🧱 Basic idea: get to hubs first
(airline networks).



Previous work—finding short paths

- 🧱 If networks have hubs can also search well:
Adamic et al. (2001)^[1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- 🧱 Basic idea: get to hubs first
(airline networks).
- 🧱 But: hubs in social networks are limited.



Outline

PoCS, Vol. 1
Small-world
networks
48 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Small-world networks

Experiments

Theory

Generalized affiliation networks

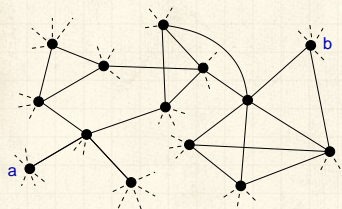
Nutshell

References



The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of **a** is closest to the target **b**?

What does 'closest' mean?

What is 'social distance'?







One approach: incorporate **identity**.



One approach: incorporate **identity**.





Identity is formed from attributes such as:

-  Geographic location
-  Type of employment
-  Religious beliefs
-  Recreational activities.



One approach: incorporate **identity**.

Identity is formed from attributes such as:





-  Geographic location
-  Type of employment
-  Religious beliefs
-  Recreational activities.

Groups are formed by people with at least one similar attribute.



One approach: incorporate **identity**.

Identity is formed from attributes such as:

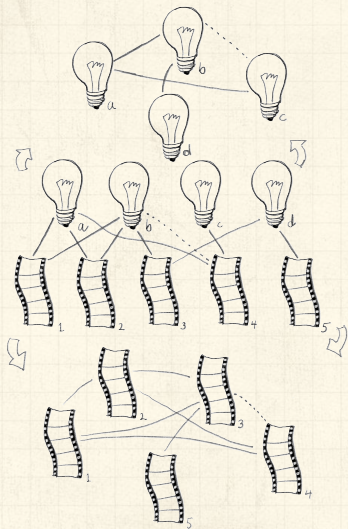
-  Geographic location
-  Type of employment
-  Religious beliefs
-  Recreational activities.

Groups are formed by people with at least one similar attribute.

Attributes \Leftrightarrow Contexts \Leftrightarrow Interactions \Leftrightarrow Networks.



Bipartite affiliation structures:



Many real-world networks have an underlying multi-partite structure.

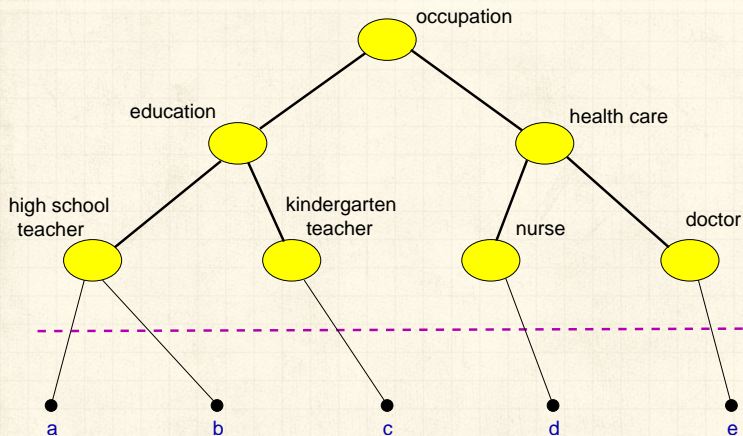
- Stories-tropes.
- Boards and directors.
- Films-actors-directors.
- Classes-teachers-students.
- Upstairs-downstairs.



Unipartite networks may be induced or co-exist.

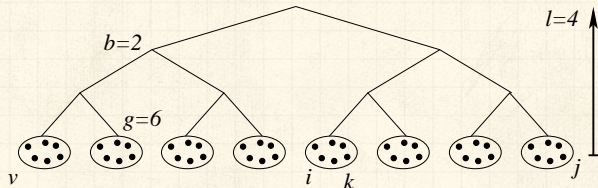


Social distance—Context distance




Models

Distance between two individuals x_{ij} is the height of lowest common ancestor.





$$x_{ij} = 3, x_{ik} = 1, x_{iv} = 4.$$



 Individuals are more likely to know each other the closer they are within a hierarchy.





 Individuals are more likely to know each other the closer they are within a hierarchy.

 Construct z connections for each node using


$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$




 Individuals are more likely to know each other the closer they are within a hierarchy.


 Construct z connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$


 $\alpha = 0$: random connections.




 Individuals are more likely to know each other the closer they are within a hierarchy.

 Construct z connections for each node using

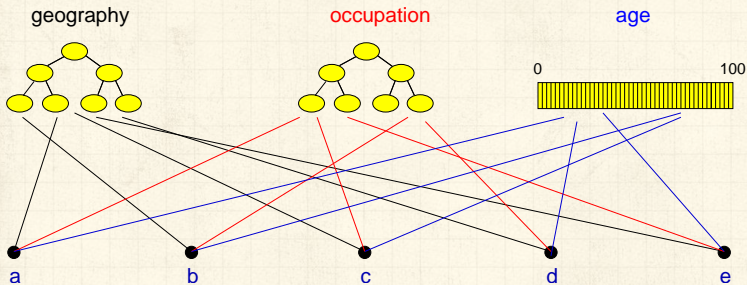
$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$


 $\alpha = 0$: random connections.

 α large: local connections.



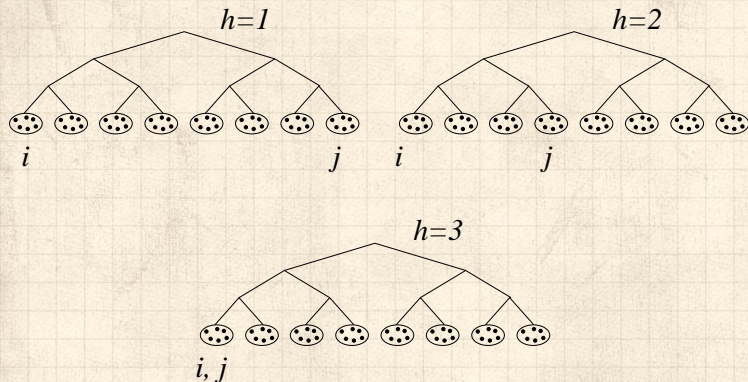
Generalized affiliation networks



 Blau & Schwartz [2], Simmel [11], Breiger [3], Watts *et al.* [13]; see also Google+ Circles.



The model



$$\vec{v}_i = [1 \ 1 \ 1]^T, \vec{v}_j = [8 \ 4 \ 1]^T$$

$$x_{ij}^1 = 4, x_{ij}^2 = 3, x_{ij}^3 = 1.$$

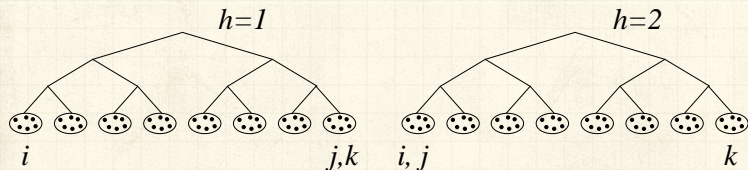
Social distance:

$$y_{ij} = \min_h x_{ij}^h.$$



The model

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$



The model

PoCS, Vol. 1
Small-world
networks
58 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Individuals know the identity vectors of



The model

PoCS, Vol. 1
Small-world
networks
58 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Individuals know the identity vectors of

1. themselves,



The model

PoCS, Vol. 1
Small-world
networks
58 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Individuals know the identity vectors of

1. themselves,
2. their friends,



The model

PoCS, Vol. 1
Small-world
networks
58 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References



Individuals know the identity vectors of

1. themselves,
2. their friends,
and
3. the target.



The model



Individuals know the identity vectors of

1. themselves,
2. their friends,
and
3. the target.



Individuals can estimate the social distance
between their friends and the target.



The model



Individuals know the identity vectors of

1. themselves,
2. their friends,
and
3. the target.



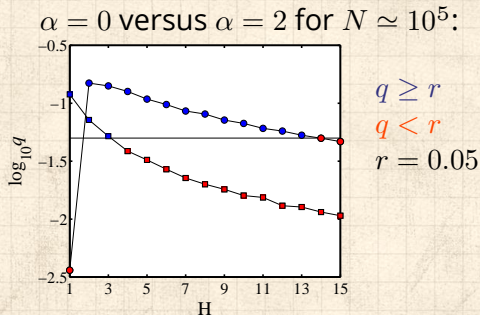
Individuals can estimate the social distance between their friends and the target.






Use a greedy algorithm + allow searches to fail randomly.



The model-results—searchable networks



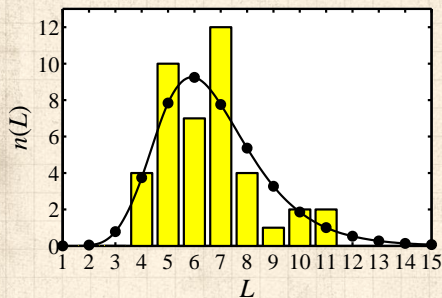
q = probability an arbitrary message chain reaches a target.

-  A few dimensions help.
-  Searchability decreases as population increases.
-  Precise form of hierarchy largely doesn't matter.





The model-results


Milgram's Nebraska-Boston data:





Model parameters:


 $N = 10^8,$

 $z = 300, g = 100,$

 $b = 10,$

 $\alpha = 1, H = 2;$

 $\langle L_{\text{model}} \rangle \simeq 6.7$

 $L_{\text{data}} \simeq 6.5$



Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.



Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.



Social Search—Real world uses

PoCS, Vol. 1
Small-world
networks
62 of 70

Small-world
networks









Experiments

Theory

Generalized affiliation
networks

Nutshell

References

-  Tags create identities for objects
-  Website tagging: bitly.com 
-  (e.g., Wikipedia)
-  Photo tagging: flickr.com 
-  Dynamic creation of metadata plus links between information objects.
-  Folksonomy: collaborative creation of metadata



Social Search—Real world uses

PoCS, Vol. 1
Small-world
networks
63 of 70

Small-world
networks

Experiments


Theory

Generalized affiliation
networks

Nutshell

References

Recommender systems:

-  Amazon uses people's actions to build effective connections between books.



Social Search—Real world uses

PoCS, Vol. 1
Small-world
networks
63 of 70

Small-world
networks

Experiments



Theory

Generalized affiliation
networks

Nutshell

References

Recommender systems:

-  Amazon uses people's actions to build effective connections between books.
-  Conflict between 'expert judgments' and tagging of the hoi polloi.



Outline

PoCS, Vol. 1
Small-world
networks
64 of 70

Small-world
networks

Experiments

Theory

Generalized affiliation
networks

Nutshell

References

Small-world networks

Experiments

Theory


Generalized affiliation networks

Nutshell

References



Nutshell for Small-World Networks:

 Bare networks are typically unsearchable.



Nutshell for Small-World Networks:

- ❏ Bare networks are typically unsearchable.
- ❏ Paths are findable if nodes understand how network is formed.



Nutshell for Small-World Networks:

- ❏ Bare networks are typically unsearchable.
- ❏ Paths are findable if nodes understand how network is formed.
- ❏ Importance of identity (interaction contexts).



Nutshell for Small-World Networks:

- ❏ Bare networks are typically unsearchable.
- ❏ Paths are findable if nodes understand how network is formed.
- ❏ Importance of identity (interaction contexts).
- ❏ Improved social network models.



Nutshell for Small-World Networks:

- ❏ Bare networks are typically unsearchable.
- ❏ Paths are findable if nodes understand how network is formed.
- ❏ Importance of identity (interaction contexts).
- ❏ Improved social network models.
- ❏ Construction of peer-to-peer networks.



Nutshell for Small-World Networks:

- ❏ Bare networks are typically unsearchable.
- ❏ Paths are findable if nodes understand how network is formed.
- ❏ Importance of identity (interaction contexts).
- ❏ Improved social network models.
- ❏ Construction of peer-to-peer networks.
- ❏ Construction of searchable information databases.



Neural reboot (NR):

Food-induced happiness

PoCS, Vol. 1
Small-world
networks
66 of 70

Small-world
networks

Experiments


Theory

Generalized affiliation
networks




Nutshell

References







https://www.youtube.com/watch?v=vC8gJ0_9o4M?rel=0 

References I

- [1] L. Adamic, R. Lukose, A. Puniyani, and B. Huberman.
Search in power-law networks.
[Phys. Rev. E, 64:046135, 2001. pdf](#) 
- [2] P. M. Blau and J. E. Schwartz.
[Crosscutting Social Circles](#).
Academic Press, Orlando, FL, 1984.
- [3] R. L. Breiger.
The duality of persons and groups.
[Social Forces, 53\(2\):181–190, 1974. pdf](#) 
- [4] S. Carmi, S. Carter, J. Sun, and D. ben Avraham.
Asymptotic behavior of the Kleinberg model.
[Phys. Rev. Lett., 102:238702, 2009. pdf](#) 



References II

- [5] C. C. Cartoza and P. De Los Rios.
Extended navigability of small world networks:
Exact results and new insights.
[Phys. Rev. Lett., 2009:238703, 2009. pdf](#) 
- [6] P. S. Dodds, R. Muhamad, and D. J. Watts.
An experimental study of search in global social
networks.
[Science, 301:827–829, 2003. pdf](#) 
- [7] M. Granovetter.
The strength of weak ties.
[Am. J. Sociol., 78\(6\):1360–1380, 1973. pdf](#) 
- [8] J. Kleinberg.
Navigation in a small world.
[Nature, 406:845, 2000. pdf](#) 





References III

- [9] G. Pickard, W. Pan, I. Rahwan, M. Cebrian, R. Crane, A. Madan, and A. Pentland. Time-critical social mobilization. [Science](#), 334:509–512, 2011. [pdf](#) ↗
- [10] M. R. Roberson and D. ben Avraham. Kleinberg navigation in fractal small world networks. [Phys. Rev. E](#), 74:017101, 2006. [pdf](#) ↗
- [11] G. Simmel. The number of members as determining the sociological form of the group. I. [American Journal of Sociology](#), 8:1–46, 1902.



References IV

- [12] J. Travers and S. Milgram.
An experimental study of the small world
problem.
[Sociometry](#), 32:425–443, 1969. pdf 
- [13] D. J. Watts, P. S. Dodds, and M. E. J. Newman.
Identity and search in social networks.
[Science](#), 296:1302–1305, 2002. pdf 
- [14] D. J. Watts and S. J. Strogatz.
Collective dynamics of ‘small-world’ networks.
[Nature](#), 393:440–442, 1998. pdf 