Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2014
Assignment 7 • code name: Crivens!

Dispersed: Friday, March 21, 2014.
Due: By start of lecture, 2:30 pm, Thursday, April 10, 2014.
Some useful reminders:
Instructor: Peter Dodds
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Office hours: 3:45 pm to 4:15 pm, Tuesday, and 12:45 pm to 2:15 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

For questions 1–6:

Consider the simple spreading mechanism on generalized random networks for which each link has a probability $\beta \le 1$ of successfully transmitting a disease.

We assume that this transmission probability is tested only once: either a link will or will not be able to send an infection one way or the other (this is a bond percolation problem). We'll call these edges 'active.'

Denote the degree distribution of the network as P_k and the corresponding generating function as F_P. In class, we wrote down the probability that a node has k active edges as

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

1. Given a random network with degree distribution P_k, find $F_{\tilde{P}}$, the generating function for $\tilde{P}_k$, in terms of F_P.

2. Find the generating function for $\tilde{R}_k$, the analogous version of R_k, the probability that a random friend has k other friends.

3. For standard random (ER) networks, use your results from the preceding questions to find the critical value of $\langle k \rangle$ above which global spreading occurs.

4. Find an expression connecting the three quantities β, the average degree $\langle k \rangle$, and the size of the giant component $\tilde{S}_1$.

5. What is the slope of the $\tilde{S}_1$ curve near the critical point for ER networks?

6. Using whichever method you find most exciting, plot how $\tilde{S}_1$ depends on $\langle k \rangle$ for $\beta = 1$, $\beta = 0.8$, and $\beta = 0.5$.

7. **Zombies!**

   (Optional. But taking practical precautions for your survival in the event of a global zombie attack is not optional.)

   Network version of the SZR model:

   Based on the work of Munz *et al.* [1], we will model Zombie attacks on generalized random networks (the paper is here).

   There are three states: S, susceptible, Z, zombie, and, R, removed.

   For the random mixing model studied by Munz et al., the differential equations are

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \theta - \beta S Z - \delta S,$$
$$\frac{\mathrm{d}Z}{\mathrm{d}t} = \beta S Z + \zeta R - \alpha S Z,$$
$$\text{and } \frac{\mathrm{d}R}{\mathrm{d}t} = \delta S + \alpha S Z - \zeta R,$$

   where

   θ is the birth rate of new susceptibles;

   β is the rate at which susceptibles who bump into zombies become zombies

   δ is the background, non-zombie related death rate for susceptibles;

   ζ is the rate at which the dead (removed) are resurrected as zombies;

   and α is the rate at which susceptibles defeat zombies (through traditional methods shown in movies).

   For our purposes, consider a random network with degree distribution P_k containing completely susceptible individuals and discrete time updates. We'll now think of the parameters above as probabilities, and ignore birth and death processes ($\theta = \delta = 0$).

   We'll further assume that if a susceptible takes out a zombie, the latter cannot resurrect. So this means there's a fourth category, let's call it D for definitely dead.

   Assume that in each time step, all edges convey interactions, meaning each individual interacts with each of their neighbors.

Under what conditions (P_k and spreading parameters) will local zombification be guaranteed to take off (i.e., grow exponentially, at least in the short term), given one randomly chosen individual becomes the first zombie?

(The long term dynamics will likely be complicated so we will focus on the initial dynamics.)

See http://www.wired.com/wiredscience/2009/08/zombies/ for more information/enjoyment.

References

[1] P. Munz, I. Hudea, J. Imad, and R. J. Smith? When zombies attack!: Mathematical modelling of an outbreak of zombie infection. In J. M. Tchuenche and C. Chiyaka, editors, *Infectious Disease Modelling Research Progress*, pages 133–150. Nova Science Publishers, Inc., 2009. pdf