Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2014
Assignment 4 • code name: H.M.S. Pinafore

Dispersed: Tuesday, February 18, 2014.
Due: By start of lecture, 2:30 pm, Thursday, March 13, 2014.
Some useful reminders:
Instructor: Peter Dodds
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E-mail: peter.dodds@uvm.edu
Office hours: 3:45 pm to 4:15 pm, Tuesday, and 12:45 pm to 2:15 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

Size-density laws:

1. For a uniformly distributed population, to minimize the average distance between individuals and their nearest facility, we've made a claim that facilities would be placed at the centres of the tiles on a hexagonal lattice (or the vertices of a triangular lattice). Why is this?

2. In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent ([1]; also see Supply Networks lecture notes).

   Repeat this argument for the d-dimensional case and find the general form of the exponent β in $D \propto \rho^{\beta}$.

3. Following Um et al.'s approach [2], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the cost function:

   $$c_i = n_i \langle r_i \rangle^{\beta} \text{ with } 0 \leq \beta \leq 1,$$

   where, respectively, n_i and $\langle r_i \rangle$ are population and the average 'source to sink' distance for the population of the ith Voronoi cell (which surrounds the ith facility).

   Note that $\beta = 0$ corresponds to purely commercial facilities, and $\beta = 1$ to strongly social ones.

References

[1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. *Phys. Rev. E*, 74:016117, 2006. pdf

[2] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. *Proc. Natl. Acad. Sci.*, 106:14236–14240, 2009. pdf