Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2014
Assignment 2 • code name: Potoroo

Dispersed: Thursday, January 30 (Telethermish), 2014.
Due: By start of lecture, 2:30 pm, Thursday, February 13, 2014.
Some useful reminders:
Instructor: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 3:45 pm to 4:15 pm, Tuesday, and 12:45 pm to 2:15 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. Data snaring and wrangling:

   Find two (2) interesting, large network data sets online. The networks may be weighted or not, directed or undirected.

   Transform each network's representation into row, column, and weight vectors as per the first assignment. The row vector contains the node at the start of an edge, the column vector the ends, and the weights, well, the weight of the edge.

   Include a one line description for each network along with a link to the data source.

   Please submit your data via email with the subject heading
   "CoNKs: Network submission for Assignment 2".

   In the next assignment, we'll examine all submitted networks.

2. Tokunaga's law is statistical but we can consider a rigid version. Take $T_1 = 2$ and $R_T = 2$ and draw an example network of order $\Omega = 4$ with these parameters.

   Please take some effort to make your network look somewhat like a river network.

3. Show $R_s = R_\ell$. In other words show that Horton's law of stream segments matches that of main stream lengths.

4. Tokunaga's law implies Horton's laws:

In lectures, we established the following:

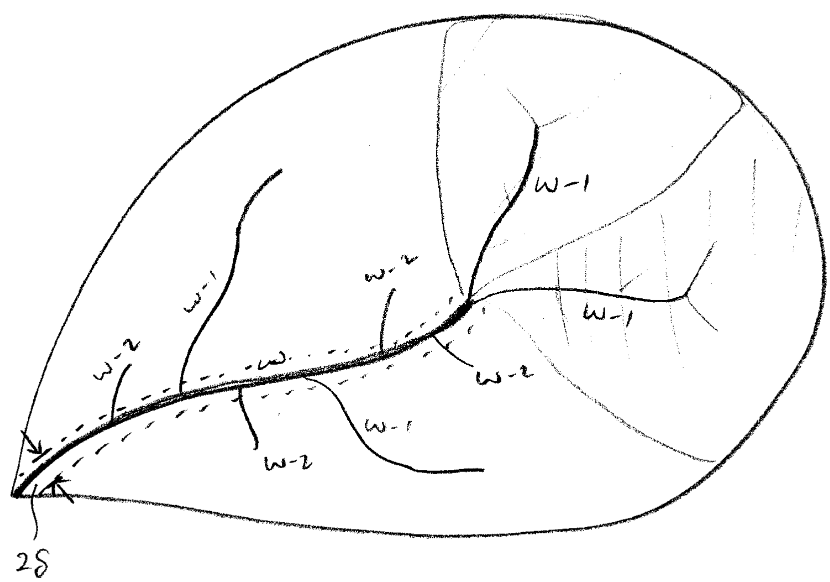
$$n_\omega = \underbrace{2\, n_{\omega+1}}_{\text{generation}} + \sum_{\omega'=\omega+1}^{\Omega} \underbrace{T_{\omega'-\omega}\, n_{\omega'}}_{\text{absorption}}$$

From here, derive Horton's law for stream numbers: $n_\omega / n_{\omega+1} = R_n$, where $R_n > 1$ and is independent of ω, and find R_n in terms of Tokunaga's two parameters T_1 and R_T.

5. Show $R_n = R_a$ by using Tokunaga's law to find the average area of an order ω basin, $\bar{a}_\omega$, in terms of the average area of basins of order 1 to $\omega - 1$.

   (In lectures, we use Horton's laws to roughly demonstrate this result.)

   Here's the set up:



   Using the Tokunaga picture, we see a basin of order ω can be broken down into non-overlapping sub-basins.

   Connect $\bar{a}_\omega$ to the average areas of basins of lower orders as follows:

$$\bar{a}_\omega = 2\bar{a}_{\omega-1} + \sum_{\omega'=1}^{\omega-1} T_{\omega,\omega'} \bar{a}_{\omega'} + 2\delta \bar{s}_\omega.$$

   The first term on the right hand side corresponds to the two 'generating' streams of order $\omega - 1$. The second term (the sum) accounts for side streams entering the sole order ω stream segment in the basin. And the last term gives the contribution of 'overland flow,' i.e., flow that does not arrive in the main stream segment through a stream. The length scale δ is the typical distance from stream to ridge.

6. For river networks, basin areas are distributed according to $P(a) \propto a^{-\tau}$.
   Determine the exponent τ in terms of the Horton ratios R_n and R_s.