

When



Slide Rules Ruled

Before electronic calculators, the mechanical slide rule dominated scientific and engineering computation

By Cliff Stoll

Two generations ago a standard uniform identified engineers: white shirt, narrow tie, pocket protector and slide rule. The shirt and tie evolved into a T-shirt sporting some software advertisement. The pocket protector has been replaced by a cell phone holster. And the slide rule has become an electronic calculator.

Take another look at that slide rule. Pull it out of the drawer you stashed it in 30 years ago or make one of your own [see box on next page]. You'll see why it was once so valuable.

Before the 1970s the slide rule, or slipstick, was as common as the typewriter or the mimeograph machine. A few seconds of fiddling let scientists and engineers multiply, divide and find square and cube roots. With a bit more effort, techies could also compute ratios, inverses, sines, cosines and tangents.

Inscribed with a dozen or more function scales, the slide rule symbolized the mysteries of arcane science. Truth is, though, two scales did most of the work, as many technical jobs boiled down to multiplication and division. A pianist might play most of the ivories on the keyboard, but rarely did any engineer use all the scales on his (almost never her) slide rule.

Some engineers, perhaps bucking for promotion, wielded slide rules made of exotic mahogany and boxwood; others sported rules fashioned from ivory, aluminum or fiberglass. Cheapskates—including this author—carried plastic ones. From the finest to the humblest, however, all slide rules were based on logarithms [see box on page 85].

Birth of the Slide Rule

JOHN NAPIER, a Scottish mathematician, physicist and astronomer, invented logarithms in 1614. His Canon of Logarithms begins: "Seeing there is nothing that is so troublesome to mathematical practice, nor doth more molest and hinder calculators, than the multiplications, divisions, square and cubical



DO-IT-YOURSELF SLIDE RULE

You can build a working slide rule from paper and cellophane tape. Photocopying these plans onto thicker paper yields a reasonably robust calculating instrument. These construction plans are also available at www.sciam.com/ontheweb

a Cut here

ASSEMBLY INSTRUCTIONS

CHRIS HAMANN AND NANCY SHAW

- 1** Cut out the entire white panel (a). Cut along line between parts A and B (b), then remove excess (c).
- 2** Fold part A along the dotted lines.
- 3** Slip part B into the folded part A.
- 4** To make the cursor (the sliding window that is inscribed with a hairline), use the guides to the left to measure two pieces of transparent tape. Make one section the length of the black line and the other the length of the red line. Place the adhesive sides together.
- 5** Draw a line with a fine marker in the middle.
- 6** Wrap the folded tape around the slide rule for sizing. Use the adhesive end to complete the cursor. Slide cursor onto the rule.

b Cut here

Part A

Part B
(Slider)

Left-hand index →

→ Right-hand index

HOW TO USE A SLIDE RULE

First, get your hands on a slide rule. The top stationary scale usually has the A scale; the B and C scales reside on the central slider. The D scale sits on the bottom stationary scale. The left-hand index is on the slider—it is the farthest left digit 1 on the C scale. At the extreme right of the slider, you will find another number 1 on the C scale—that is the right-hand index. Finally, the mobile cursor contains the hairline.

To multiply two numbers, move the slider until the left-hand index points to the first number on the D scale. Now slide the cursor hairline over so it points to the second number on the C scale. The answer will appear under the hairline on the D scale. So to multiply 2 times 4, adjust the C scale until the left-hand index points to 2 on the D scale. Move the hairline to rest over the 4 on the C scale. You'll find the answer, 8, right under the hairline on the D scale.

If your calculation extends off the end of the slide rule, use the right-hand index. So to multiply 7 times 6, set the right-hand index over 7 on the D scale and the hairline over 6 on the C scale. Read 4.2 on the D scale and then remember that the slippery decimal point must be shifted one place to the right to give the correct answer, 42.

To divide, set the hairline over the dividend on the D scale. Then shift the slider until the divisor lies under the hairline (and right next to the dividend). The quotient will be under the index. For example, let's divide 47 by 33. Move the cursor so that the hairline points to 4.7 on the D scale. Move the slider until 3.3 on the C scale rests under the hairline. Now the left-hand index sits adjacent to the answer, 1.42.

Want to find the square of a number? You will not need to move the slider. Just place the hairline over a number on the D scale. Look up at the A scale, where the hairline points to the square. So, right above 7 on the D scale, you will find 4.9 on the A scale. Slip the decimal point to the right to get the answer, 49.

To determine square roots, there is no need to move the slider. But notice that the A scale is divided into two parts: the left half runs from 1 to 10, and the right half goes from 10 to 100. To find the square root of any number between 1 and 10, place the hairline over the number on the left side of the A scale and read out the square root from the D scale. Use the right half of the A scale to take the square root of numbers between 10 and 100. When you write numbers in scientific notation, those with even exponents (such as 1.23×10^4) will be found on the left side of the A scale; those with odd exponents (such as 1.23×10^3) are on the right.

You can discover quite a few shortcuts—for instance, the cursor works as a short-term memory in chaining calculations. Or try using the CI scale to prevent calculations from running off the end of the slipstick.

You will find additional scales on your homemade slide rule. The K scale is used for cubes and cube roots; the S and T scales give sines and tangents. The L scale gives the logarithm of a number on the D scale.

Try these on your homemade slipstick. With a bit of practice, you may be surprised at its ease of use and its utility. —C.S.

extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.”

Yes, logarithms—that horror of high school algebra—were actually created to make our lives easier. In a few generations, people will be equally shocked to learn that computers were also invented to make our lives easier.

So how did Napier's logarithms work? Listen to the inventor: “Cast away from the work itself even the very numbers themselves that are to be multiplied, divided, and resolved into roots, and putteth other numbers in their place which perform much as they can do, only by addition and subtraction, division by two or division by three.”

Which is to say that by using logs, multiplication simplifies into sums, division becomes subtraction, finding a square root turns into dividing by two, and figuring a cube root becomes dividing by three. For example, to multiply 3.8 by 6.61, you look up the logarithms of those numbers in a table. There you will find 0.58 and 0.82. Add these together to get 1.4. Now go back to the table and find the number whose log is 1.4 to get a close approximation of the answer: 25.12. Begone ye slippery errors!

Napier's invention revolutionized mathematics—mathematicians adopted it immediately to speed their calculations. German astronomer Johannes Kepler used these modern logarithms to calculate the orbit of Mars at the start of the 17th century. Without their assistance, he might never have discovered his three laws of celestial mechanics.

Henry Briggs, England's most eminent mathematician of the time, traveled to Scotland just to meet Napier. He introduced himself with: “My lord, I have undertaken this long journey purposefully to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help unto



SLIDE RULE was invented by English minister William Oughtred in 1622. Robert Bissaker constructed the first slide rule whose slide fit into a slot in a fixed stock in 1654 (bottom). Henry Sutton's slide rule (top) was an early circular rule (circa 1663).

astronomy ... I wonder why nobody else found it out before, when, now being known, it appears so easy." Briggs recognized genius; Napier went on to invent the decimal point and calculating rods (known as Napier's bones) and to lay the groundwork for Isaac Newton's calculus.

Napier had simplified computational tasks, but ready access to books of log tables was crucial to the procedure. So in 1620 mathematician Edmund Gunter of London marked a ruler with logarithms, which let his calculating colleagues find logs without a trip to the library. Gunter drew a number line in which the positions of numbers were proportional to their logs. In his scale, succeeding numbers are spread out on the left and squashed together at the right end. Two numbers could now be multiplied by measuring the distance from the beginning of the scale to one factor with a pair of dividers, then moving them to start at the other factor and reading the number at the combined distance.

Around 1622 William Oughtred, an Anglican minister in England, placed two sliding wooden logarithmic scales next to each other and created the first slide rule. A few years later he made a circular slide rule. Not that Oughtred crowed about his achievements. As one who loved pure mathematics, he

“ Multiplication just required lining up two numbers and reading a scale. ”

probably felt that his invention was not worth much. After all, mathematicians created equations; they did not apply them. (This is still true today: making money often means finding an application for what someone else has developed.)

For whatever reason, Oughtred failed to publish news of his invention, but one of his students, Richard Delamain, claimed in a 1630 pamphlet to have come up with the circular slide rule. More engineer than mathematician, Delamain was delighted with its portability, writing that it was “fit for use on Horse backe as on Foote.”



OTIS KING, a London engineer, wrapped several feet of scales around a pocket-size cylinder in 1921 to achieve a portable slide rule with impressive resolution.

Denied credit for his invention, Oughtred was outraged. He rallied his friends, who accused Delamain of “shamelessness” and being the “pickpurse of another man’s wit.” The argument would continue until Delamain’s death, serving neither man much good. Oughtred later wrote, “This scandall hath wrought me much prejudice and disadvantage.”

Look Ma, No Logs!

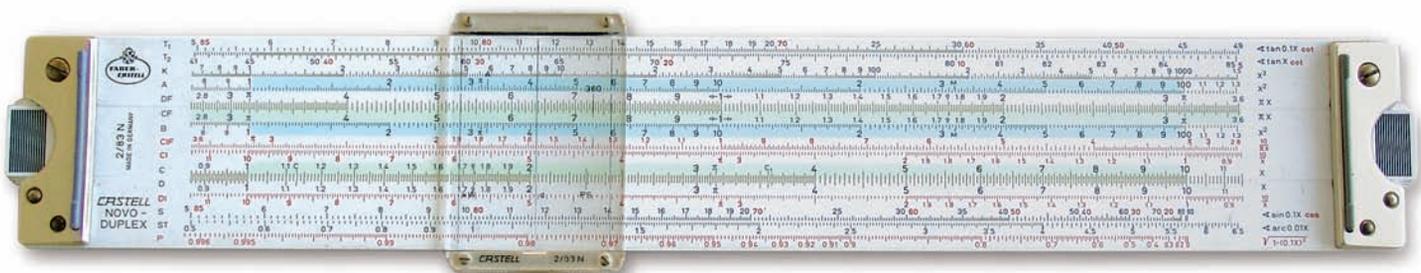
WITH OUGHTRED’S INVENTION in hand, no one needed a book of logarithms or even had to know what a log was. Multiplication just required lining up two numbers and reading a scale. It was quick and eminently portable. The slide rule would automatically “cast away numbers.”

A wonderful idea, yet slide rules took two centuries to catch on. As late as 1850, British mathematician Augustus De Morgan lamented the resistance: “For a few shillings, most persons might put into their pockets some hundred times as much power of calculation as they have in their heads.”

The slide rule was improved and extended during the first half of the 1800s. In a lecture before the Royal Society in 1814, Peter Roget (the creator of the thesaurus) described his invention, the log-log slide rule. With this tool, he could easily calculate fractional powers and roots, such as 30.6 to the 2.7th power. The utility of the log-log rule, however, was not appreciated until 1900, when chemists, electrical engineers and physicists began to face increasingly complex mathematics.

It took a 19-year-old French artillery lieutenant—Amédée Mannheim—to popularize the slide rule. In 1850 he chose the four most useful scales and added a movable cursor (a sliding pointer to line up numbers on the scales). Within a few years the French army adopted his device. When the Prussian infantry is attacking, who has time to aim a cannon using long division?

In time, European engineers, surveyors, chemists and astronomers carried Mannheim’s improved slide rule. After World War I, American scientists began to adopt them. All



FABER-CASTELL 2/83N slide rule is considered by some to be the finest and most beautiful slide rule ever made.

WWW.VINTAGECALCULATORS.COM [top]; FABER-CASTELL AG [bottom]

but the cheapest slide rules displayed squares and roots; most also computed cubes, cube roots, inverses, sines and tangents. Sophisticated ones might include hyperbolic functions to let electrical engineers calculate vectors or help structural engineers find the shape of catenary curves, which are important elements in suspension bridges, for instance. To pry more precision out of their slipsticks, manufacturers added magnifiers to better judge positions on scales, inscribed ever finer tick marks and built longer slide rules. They mapped Napier's logarithms onto circles, spirals, disks and cylinders.

In 1921 London engineer Otis King spiraled a five-foot-long logarithmic scale around an inch-diameter cylinder that could fit in a pocket. Engineers marveled at its four digits of precision. For even more exactitude, a scientist might invest in Fuller's Rule, the granddaddy of high-precision slide rules. A 41-foot logarithmic helix snakes around the exterior of a foot-long cylinder; by using a special indicator, it gives the precision of an 83-foot scale, letting users do arithmetic with five digits of resolution. The elaborate contraption might be mistaken for an engraved rolling pin.

With few alternatives, techies adapted to slipsticks. In turn, slide-rule makers inscribed additional marks to speed calculations. Typically you could find pi, pi/4, the constant e (the base of "natural" logarithms) on the scales, and occasionally cursor marks to convert inches to centimeters or horsepower to watts. Specialized slide rules appeared with molecular weights for chemists, hydraulic relations for shipbuilders and radioactive decay constants for atom bomb designers.

By 1945 the log-log duplex slide rule had become ubiquitous among engineers. With nearly a dozen scales on each side, it would let users raise a number to an arbitrary power as well as handle sines, cosines and hyperbolic trigonometry functions with ease. During World War II, American bombardiers and navigators who required quick calculations often used specialized slide rules. The U.S. Navy designed a generic slide rule "chassis," with an aluminum body and plastic cursor, into which celluloid cards could be inserted for specialized calculations of aircraft range, fuel use and altitude.

By the 1960s you could not graduate from engineering school without a week's instruction in the use of a slipstick. Leather-cased slide rules hung from belts in every electrical engineering department; the more fashionable sported slide-rule tie clips. At seminars, you could tell who was checking the speaker's numbers. High-tech firms gave away slide rules imprinted with company trademarks to customers and prospective employees.

High Noon for the Slipstick

CONSIDER THE ENGINEERING achievements that owe their existence to rubbing two sticks together: the Empire State Building; the Hoover Dam; the curves of the Golden Gate Bridge; hydromatic automobile transmissions; transistor radios; the Boeing 707 airliner. Wernher Von Braun, the designer of the German V-2 rocket and the American Saturn 5 booster, relied on a rather plain slide rule manufactured by the German com-

Logarithm Log

A bit fuzzy about logarithms? Here is a short summary: If $a^x = m$, then x , the exponent, can be said to be the logarithm of m to the base a . Although a can be any number, let us focus on common logarithms, or the logs of numbers where $a = 10$. The common log of 1,000 is 3 because raising 10 to the third power, 10^3 , is 1,000. Conversely, the antilog of 3 is 1,000; it is the result of raising 10 to the third power.

Exponents do not have to be integers; they can be fractions. For example, $10^{0.25}$ equals about 1.778, and $10^{3.7}$ equals about 5,012. So the log of 1.778 is 0.25, and the log of 5,012 is 3.7.

When you express everything in terms of 10 to a power, you can multiply numbers by just adding the exponents. So $10^{0.25}$ times $10^{3.7}$ is $10^{3.95}$ ($10^{0.25+3.7}$). What does $10^{3.95}$ equal? Look up the antilog of 3.95 in a log table, and you'll find 8,912, which is indeed about equal to the product of 1.778 and 5,012. (Common logs can be found by entering "log (x)" into Google, for example, or by consulting log tables in libraries.)

Just as multiplication simplifies to addition, division becomes subtraction. Here is how to divide 759 by 12.3 using logs. Find the logs of 759 and 12.3: 2.88 and 1.09. Subtract 1.09 from 2.88 to get 1.79. Now look up the antilog of 1.79 to get the answer, 61.7.

Need to calculate the square root of 567.8? Just determine its log: 2.754. Now divide that by 2 to get 1.377. Find the antilog of 1.377 for the answer: 23.82.

Naturally, complications arise. Log tables list only the mantissa—the decimal part of the log. To get the true logarithm, you must add an integer (called the characteristic) to the mantissa. The characteristic is the number of decimal places to shift the decimal point of the associated number. So to find the log of 8,912, you would consult a log table and see that the log of 8.912 is 0.95. You would then determine the characteristic of 8,912, which is 3 (because you must shift the decimal point three places to the left to get from 8,912 to 8.912). Adding the characteristic to the mantissa yields the true common log: 3.95.

Because common logs are irrational (a number expressed as an infinite decimal with no periodic repeats) and log tables have limited precision, calculations using logs can provide only close approximations, not exact answers.

Logarithms show up throughout science: Chemists measure acidity using pH, the negative log of a liquid's hydrogen ion concentration. Sound intensity in decibels is 10 times the log of the intensity divided by a reference intensity. Earthquakes are often measured on the Richter scale, which is built on logarithms, as are the apparent brightness magnitudes of stars and planets.

Finally, logs pop up in everyday usage. Many graphs that depict large numbers employ logarithmic scales that map numbers by orders of magnitude (10, 100, 1,000 and so forth)—the same scales that appear on slide rules. —C.S.



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"COMPUTERS" used to refer to humans who spent their time calculating numbers. This 1953 advertisement foreshadowed the changes that would occur when electronic calculators and digital computers began to come into more common use.

pany Nestler. Indeed, the Pickett company made slide rules that went on several Apollo missions, as backup calculators for moon-bound astronauts. Soviet engineer Sergei Korolev used a Nestler rule when he designed the Sputnik and Vostok spacecraft. It was also the favorite of Albert Einstein.

THE AUTHOR

CLIFF STOLL is best known for breaking up a ring of hackers during the early days of the Internet, as described in his book *The Cuckoo's Egg*. His other books are *High Tech Heretic: Why Computers Don't Belong in the Classroom* and *Silicon Snake Oil*. Stoll received a doctorate in planetary science from the University of Arizona and now makes Klein bottles and teaches physics to eighth graders. In his previous life, he worked at the Space Telescope Science Institute, the Purple Mountain Observatory in China, the Kitt Peak National Observatory, the Keck Observatory and the Harvard-Smithsonian Center for Astrophysics. Stoll and his wife live in Oakland, Calif.; the log of the number of their children is about 0.301, and they have almost $10^{0.4772}$ cats. The author would like to thank Regina McLaughlin, Bob Otnes and Walter Shawlee for their help with this article.

Yet slide rules had an Achilles' heel; standard models could typically handle only three digits of precision. Fine when you are figuring how much concrete to pour down a hole but not good enough for navigating the path of a trans-lunar space probe. Worse yet: you have to keep track of the decimal place. A hairline pointing to 3.46 might also represent 34.6, 3,460 or 0.00346.

That slippery decimal place reminded every competent engineer to double-check the slide rule's results. First you would estimate an approximate answer and then compare it with the number under the cursor. One effect was that users felt close to the numbers, aware of rounding-off errors and systematic inaccuracies, unlike users of today's computer-design programs. Chat with an engineer from the 1950s, and you will most likely hear a lament for the days when calculation went hand-in-hand with deeper comprehension. Instead

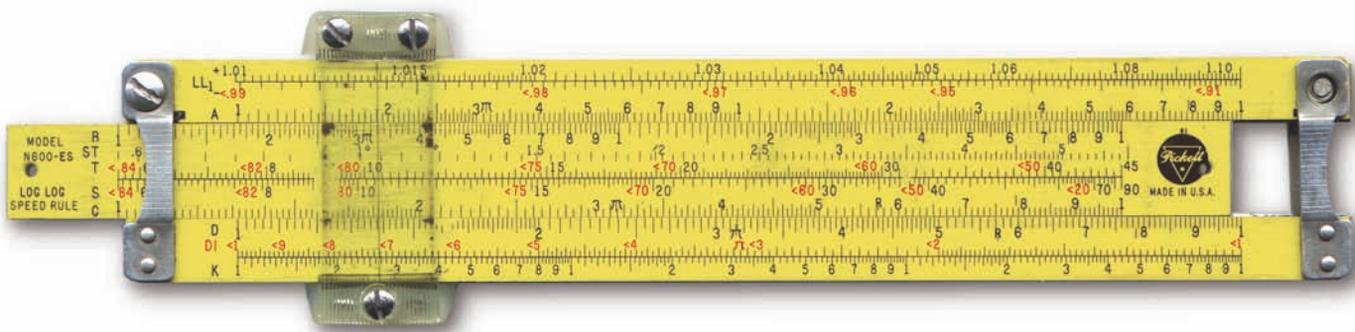
“The slide rule helped to design the very machines that would render it obsolete.”

of plugging numbers into a computer program, an engineer would understand the fine points of loads and stresses, voltages and currents, angles and distances. Numeric answers, crafted by hand, meant problem solving through knowledge and analysis rather than sheer number crunching.

Still, with computation moving literally at a hand's pace and the lack of precision a given, mathematicians worked to simplify complex problems. Because linear equations were friendlier to slide rules than more complex functions were, scientists struggled to linearize mathematical relations, often sweeping high-order or less significant terms under the computational carpet. So a car designer might calculate gas consumption by looking mainly at an engine's power, while ignoring how air friction varies with speed. Engineers developed shortcuts and rules of thumb. At their best, these measures led to time savings, insight and understanding. On the downside, these approximations could hide mistakes and lead to gross errors.

Because engineers relied on imperfect calculations, they naturally designed conservatively. They made walls thicker than need be, airplane wings heavier, bridges stronger. Such overengineering might benefit reliability and durability, but it cost dearly in overconstruction, poorer performance and sometimes clumsy operation.

The difficulty of learning to use slide rules discouraged their use among the hoi polloi. Yes, the occasional grocery store manager figured discounts on a slipstick, and this author once caught his high school English teacher calculating stats for trifecta horse-race winners on a slide rule during study



PICKETT N600-ES slide rule traveled with the NASA Apollo spacecraft to the moon as a backup calculator.

hall. But slide rules never made it into daily life because you could not do simple addition and subtraction with them, not to mention the difficulty of keeping track of the decimal point. Slide rules remained tools for techies.

The Fall of the Slide Rule

FOR THE FIRST HALF of the 20th century, gear-driven mechanical calculators were the main computational competitors to slide rules. But by the early 1960s, electronics began to invade the field. In 1963 Robert Ragen of San Leandro, Calif., developed the Friden 130—one of the first transistorized electronic calculators. With four functions, this desktop machine amazed engineers by silently calculating to 12 digits of precision. Ragen recalls designing this electronic marvel entirely with analog tools: “From the transistor bias currents to the memory delay lines, I fleshed out all the circuitry on my Keuffel & Esser slide rule.” The slide rule helped to design the very machines that would ultimately render it obsolete.

By the late 1960s you could buy a portable, four-function calculator for a few hundred dollars. Then, in 1972, Hewlett-

Packard built the first pocket scientific calculator, the HP-35. It did everything that a slide rule could do—and more. Its instruction manual read, “Our object in developing the HP-35 was to give you a high-precision portable electronic slide rule. We thought you’d like to have something only fictional heroes like James Bond, Walter Mitty or Dick Tracy are supposed to own.”

Dozens of other manufacturers soon joined in: Texas Instruments called their calculator product the “Slide Rule Calculator.” In an attempt to straddle both technologies, Faber-Castell brought out a slide rule with an electronic calculator on its back.

The electronic calculator ended the slipstick’s reign. Keuffel & Esser shut down its engraving machines in 1975; all the other well-known makers—Post, Aristo, Faber-Castell and Pickett—soon followed suit. After an extended production run of some 40 million, the era of the slide rule came to a close. Tossed into desk drawers, slide rules have pretty much disappeared, along with books of five-place logarithms and pocket protectors.

Today an eight-foot-long Keuffel & Esser slide rule hangs on my wall. Once used to teach the mysteries of analog calculation to budding physics students, it harkens back to a day when every scientist was expected to be slide-rule literate. Now a surfboard-size wall hanging, it serves as an icon of computational obsolescence. Late at night, when the house is still, it exchanges whispers with my Pentium. “Watch out,” it cautions the microprocessor. “You never know when you’re paving the way for your own successor.”



HP-35 POCKET CALCULATOR sounded the death knell for the slide rule. Introduced by Hewlett-Packard in 1972, the \$395 handheld device combined large-scale integration circuits (six chips in all) with a light-emitting diode display. It was powered by batteries or an AC adapter.

MORE TO EXPLORE

A History of the Logarithmic Slide Rule and Allied Instruments. Florian Cajori. First published in 1909. Reprinted by Astragal Press, 1994.

Slide Rules: Their History, Models and Makers. Peter M. Hopp. Astragal Press, 1999.

Basic slide-rule instructions: www.hpmuseum.org/srinst.htm

Interactive slide-rule simulation: www.taswegian.com/SRTP/JavaSlide/JavaSlide.html

Peter Fox offers an explanation of logarithms and slide rules at www.eminent.demon.co.uk/sliderul.htm

The Oughtred Society, dedicated to the preservation and history of slide rules and other calculating instruments: www.oughtred.org

Slide-rule discussion forum: groups.yahoo.com/group/sliderule

Walter Shawlee’s Sphere Research’s Slide Rule Universe sells slide rules and related paraphernalia: www.sphere.bc.ca/test/sruniverse.html