UVM MS+PHD Exam topics
SYLLABUS FOR COMBINATORICS

Basic enumeration: binomial coefficients, double-counting, inclusion-exclusion, derangements, Möbius function, Möbius inversion, Burnside's lemma, Stirling numbers, Bell numbers, generating functions, Fibonacci and Catalan numbers;

Posets: posets, (anti)chains, hypercubes, Erdős-Szekeres lemma
PhD only: Dilworth's theorem, Sperner's theorem, symmetric chains, Erdős-Ko-Rado theorem, the incidence algebra of a poset;

Partitions: *PhD only:* the function $p_k(n)$, Ferrers diagram, asymptotics, Euler's identity, asymptotics;

Graph theory: (spanning) trees, paths, cycles, Hall's theorem, Caley's theorem, connectivity, vertex/edge covering, Menger's theorem, Tutte's theorem, bipartite graphs, König's theorem, Erdős-Pósa theorem, Hamiltonicity, coloring, Tutte polynomial, Turán and Ramsey numbers, flows, Ford-Fulkerson algorithm, Birkhoff's theorem, circulations, planarity, Kuratowski's theorem, the Matrix-tree theorem.
PhD only: Hadwiger conjecture, minors, well-quasi-ordering, Robertson-Seymour, tree-width, probabilistic method

Generating functions: Weighted sums of objects, Ordindary GFs, exponential GFs, Dirichlet series, exponential formula, 'Snake Oil', WZ Method.

Symmetric functions and tableaux: Standard and semistandard tableaux, tableaux insertion, hook formula, jeu de taquin, Knuth relations, Robinson-Schensted-Knuth, Littlewood-Richardson rule, Kostka numbers, standard bases of the ring of symmetric functions, various definitions of Schur functions, Jacobi-Trudi determinant, ω involution, Schur expansions of h_α and e_α.

Projective and combinatorial geometry: *PhD only:* projective and affine geometries, duality, Desargues' theorem, combinatorial geometries, geometric lattices, Greene's theorem, arcs and subplanes in projective planes

Matroids: Bases, independent sets, flats, geometric lattices, circuits, rank functions, representability, vector configurations, hyperplane arrangements, zonotopes, unimodular matroids, regular matroids, graphic matroids, transversal matroids and gammoids, minors, truncations, Tutte activities, Tutte polynomials and characteristic polynomials, independence and no broken circuits complexes, matroid polytopes, and Bergman fans.

Polytopes: Interior and exterior presentations, Farkas Lemma, Carathéodory's theorem, polarity, normal fans, faces, the f-vector, shellings, Euler-Poincaré relation, simple and simplicial polytopes, h-vectors and the Dehn-Somerville relations, the statement of the g-theorem, regular subdivisions and fiber polytopes, Gale duality and oriented matroids, zonotopes, and Steinitz theorem.