Numerical Analysis MS Qualifying Exam
University of Vermont
January 2025

Letter assigned by proctor (not your name): _____

➤ **You will have two hours to complete the exam.**
Instructions and passing criteria: No calculators or other electronic gadgets allowed. Students must complete two (2) problems and attempt one (1) additional problem. To have attempted a problem, you must correctly outline the main idea of the solution and begin the calculation, but need not finish it.
Make sure to provide explanations to all steps of your solutions and to explain all answers.

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1. (a) State the Secant method for solving the equation $f(x) = 0$ and explain how it approximates the Newton method.

   (b) Suppose that the graph of $f(x)$ is concave down near the root x_*, i.e., $f(x_*) = 0$ and $f''(x_*) < 0$. Make a sketch showing how the next iteration x_{i+1} is found from the previous iteration(s) for both (i) the Newton method and (ii) the Secant method. Based on the sketch, predict with a brief explanation which method will converge faster to x_*. In your sketch, show one iteration only.

2. Compute the quadratic polynomial interpolating the data points $(1/3, 2), (1/4, -1), (1, 7)$ using the Lagrange basis and integrate the resulting polynomial on the interval $[0, 1]$ using Simpson's rule at the nodes $x_0 = 0, x_1 = 1/2, x_2 = 1$.

3. (a) Compute a Singular Value Decomposition of the matrix A.

$$A = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

   (b) What is the best rank-1 approximation of A?

   (c) Why is this matrix from question (b) a good or poor approximation of A?

4. (a) Derive both the forward Euler (FE) and backward Euler (BE) methods of numerical integration of an ODE $y' = f(t, y)$ from Taylor Series. Draw a picture showing the geometric interpretation of each method.

   (b) Use Taylor series to derive the local truncation error of the FE method.

   (c) Suppose $f(t, y) = \lambda y$ where the constant $\lambda < 0$. Assuming a stepsize h in time, find a condition (if any) which will ensure that the method will not blow up. Obtain such condition(s) separately for the FE and BE methods.