# UVM
# Handbook
# for Graduate Studies
# in Mathematics
# 2021-2022

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This Handbook contains information about the Graduate Program in Mathematics at the University of Vermont. It is useful for potential students, current students, advisors, and professors. Comments or suggestions may be relayed to Dr. Taylor Dupuy, the Director of the Mathematics Graduate Program, Department of Mathematics and Statistics, University of Vermont, 82 University Place Burlington, VT 05405. Questions about the graduate program can be sent to Dr. Dupuy at Taylor.Dupuy@uvm.edu. The department phone number is (802) 656-2940. Additional information can be found at the dept. website at http://www.uvm.edu/math and an online version of this document can be found here.

The Department of Mathematics and Statistics offers programs towards the Master of Science (M.S.) and Doctor of Philosophy in the Mathematical Sciences (Ph.D.) degrees. We also offer an Accelerated Master's Program (A.M.P.) which combines studies towards the Bachelor of Science (B.S.) and Master of Science degrees into one 5-year program. The Statistics Program also offers Master's Degrees in Biostatistics and Statistics. This handbook does not cover those degrees.

Statistics students should contact Professor Jeff Buzas at Jeff.Buzas@uvm.edu. The curriculum for all degrees has two main streams: pure mathematics and applied mathematics, and in addition our PhD offers a stream in statistics. Students are encouraged to take courses common to both areas, enabling themto gain an appreciation of the problems and techniques in each, an understanding of the connections between them, and the common role of scientific computation.

The department is located in Innovation Hall on Central Campus. Professor Jianke Yang is the Department Chair. Senior Lecturer Helen Read is the Vice Department Chair. Professor Jeff Buzas is the Director of Statistics and as well as the Statistics Graduate Program Coordinator. Professor Taylor Dupuy is Mathematics Graduate Program Coordinator.

There is also a standing departmental Mathematics Graduate Committee in charge of overseeing the Mathematics Graduate Program. Any of the people listed above are more than willing to answer your questions about the department. Some relevant e-mail addresses are:

| | |
|---|---|
| Jianke Yang | Jianke.Yang@uvm.edu |
| Helen Read | Helen.Read@uvm.edu |
| Taylor Dupuy | Taylor.Dupuy@uvm.edu |

We want to help. Students with a question or a problem are encouraged to ask someone for help. Kiki Reno or Barbara Asiimwe, our department administrators, are always a good person to start with.

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|---|---|
| Kiki Reno | Kiki.Reno@uvm.edu |
| Barbara Asiimwe | Barbara.Asiimwe@uvm.edu |

New graduate students will need to see Kiki or Barbara for course materials including a textbook if they are teaching a course. They can also point out where the mailboxes and graduate student offices are located. They are available to take phone messages and in general help keep the department running smoothly.

Networked computers are available in the graduate student offices and at many locations around campus. All graduate students have accounts on the UVM network. These accounts are maintained by the University's Information Technology department. They provide access to the Internet and a variety of installed software. Students are encouraged to acquaint themselves with the system and its capabilities. For example, one can access computer algebra systems (Mathematica, MATLAB), packages for mathematical computing (PARI, SAGE, MATLAB), mathematical typesetting software (LaTeX), and various tools for doing mathematical research.

Each graduate student receives both a postal and an electronic mailbox which are used for Department, College, and University Communications.

The University of Vermont (UVM) is located in Burlington, a city of about 50,000 located on the shores of Lake Champlain, with views of Vermont's Green Mountains to the east and New York's Adirondack Mountains to the west. Burlington is 90 miles south of Montreal, 200 miles northwest of Boston and 300 miles north of New York City. This location affords many intellectual, cultural and recreational opportunities. Popular outdoor activities include hiking,

climbing, bicycling, in-line skating, mountain-biking, sailing, skateboarding, skiing and snowboarding.

## Mathematics Graduate Faculty and Students

The mathematics faculty at UVM has strengths in several areas (see below), and members are actively involved in research in their areas of expertise. The department and university have a friendly, collegial atmosphere in which students have ample opportunity to talk with professors on an informal basis. We strive to offer an education that is comprehensive yet tailored to the individual needs of our students. Students are invited to participate in the ongoing research seminars in number theory, applied mathematics, complex systems and combinatorics. These include participants from St. Michael's College, Middlebury, and other institutions.

A list of all mathematics and statistics faculty members is available from the department's home page. There you will also find links to personal web pages of individual faculty members and graduate students. These web pages contain much more information about the individuals including, in many cases, complete copies of their recent publications.

### The Graduate Faculty and Their Interests

| | |
|---|---|
| Spencer Backman | Algebraic and Geometric Combinatorics |
| Daniel Bentil | Applied Mathematics, Modeling, Biomathematics |
| Chris Danforth | Chaos, Mathematical Modeling, Complex Systems, Computational Social Science |
| Taylor Dupuy | Arithmetic Geometry, Differential Algebra, Applied Model Theory |
| Taras Lakoba | Stability of Numerical Methods, Perturbation Theory, Nonlinear Waves, Fiber Optics |
| Puck Rombach | Graph/Network Theory |
| Christelle Vincent | Number Theory and Arithmetic Geometry |
| Greg Warrington | Combinatorics |
| Mike Wilson | Fourier Analysis |
| Jianke Yang | Non-Linear Waves, Non-Linear Optics, Numerical Methods |
| Jun Yu | Applied Mathematics, Modeling, Biomathematics |

In addition to the mathematics faculty listed above, students may draw upon the statistics faculty: Jeff Buzas, Jim Bagrow, Bernard (Chip) Cole, Abigail Crocker, Paramita Saha Chaudhuri, Rich Single, and Jean-Gabriel Young.

Faculty with secondary appointments in our department are Peter Callas and George Pinder.

Our department also includes a number of lecturers, adjunct faculty, and emeritus faculty who can also play valuable roles in graduate education and training. See the department webpage for a complete list.

## University Information

The Department of Mathematics and Statistics (656-2940) is situated administratively in the College of Engineering and Mathematical Sciences (CEMS). College offices are located in Votey Hall.

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|---|---|
| Linda Schadler, Dean | Linda.Schaler@uvm.edu |
| Doug Dickey, Assistant Dean | Douglas.Dickey@uvm.edu |
| Jennifer Phalen, Executive Assistant to the Dean | Jennifer.Phalen@uvm.edu |
| Eric Tradup, CEMS Graduate Program Coordinator | Eric.Tradup@uvm.edu |

The Graduate College (656-3160), located in the Waterman building, coordinates graduate studies across the University. Cindy Forehand is the Dean of this College. Its staff can help resolve difficulties that cannot be managed within our department. Kimberly Hess is the Director of Admissions & Enrollment Management. Sean Milnamow is the Student Services Specialist. Along with the Mathematics Graduate Director, they will be able to help you with such things as: format of theses and dissertations, your progress through the program requirements as you near graduation, changes in your program, transfer credits, and academic standing. We encourage you to familiarize yourself with their timeline for graduation.

CEMS hosts a Graduate Student Orientation the week prior to the beginning of the fall semester. The Graduate College sponsors a Graduate Teaching Assistant of the year competition. They administer Summer Research Fellowships and Travel Mini-Grants for students to present research at professional meetings.

The Center for Teaching and Learning offers workshops on various aspects of teaching, grading and the Blackboard course management software used at UVM. These are open to teaching assistants.

A majority of our students, and about half of our undergraduate majors, are from the College of Arts and Sciences (CAS), also located in Waterman.

The Office of International Educational Services (656-4296) coordinates programs, events, and services of special interest to international students. This includes Language and Writing Workshops, assistance in academic and cultural adjustment, and immigration and employment help. Much valuable information can be found online at http://www.uvm.edu/oie/. They also assist applicants and new students with obtaining I-20 forms, Statements of Support, etc.

The *Graduate Catalogue* contains a wealth of essential information, such as degree requirements, university policies, and student rights and responsibilities. The latter includes academic honesty, sexual harassment, and grievance procedures. The catalogue is linked to the Graduate College website at http://www.uvm.edu/~gradcoll. For another view of departmental services and of our undergraduate programs you may wish to consult a copy of our Handbook for Majors, which can be found on the department website.

## Information for Applicants

### The Application Process

A student interested in graduate studies in mathematics should contact the Director of the Mathematics Graduate Program, Taylor Dupuy; his e-mail address is Taylor.Dupuy@uvm.edu. Students interested in statistics should contact the Director of the Statistics Graduate Program, Jeff Buzas; his email address is Jeff.Buzas@uvm.edu. Complete applications include a statement of purpose, three reference letters, and college transcripts. International students whose native language is not English must take the Test of English as a Foreign Language (TOEFL) exam. A TOEFL score of 90 is required for admission and 100 for Teaching Assistantship consideration.

Application forms may be found on the Graduate College's web pages at: http://www.uvm.edu/~gradcoll.

Students with prior graduate study may, with the permission of the Graduate College, transfer up to 9 hours of credit from another university, or apply up to 9 hours of UVM graduate credit towards the degree. In addition, the student may apply up to 6 extra hours for courses taken during the semester in which we act upon their application.

Decisions on admissions to both the Masters and Ph.D. Programs are made on a continuing basis; students may be admitted for the fall semester during the summer just preceding it. Students may also apply to be admitted at the beginning of a spring semester. (See the subsequent section on Funding for Graduate Students for information about applying for GTA support.

## Applying to the Ph.D. Program

Well qualified applicants with a bachelor's degree may be admitted directly into the Ph.D. program. Students may also apply directly to the Master's program. We strongly suggest that applicants to the Ph.D. program contact potential thesis advisors in the department during the application process well before admissions decisions are made. (See also the subsequent section on Concurrent Credit for M.S. and Ph.D. programs.)

## Protocol for Review of Current M.S. Students Who Apply to the Ph.D. Program

Students currently enrolled in the M.S. program who wish to apply for admission to the Ph.D. program must communicate this in writing to the Director of the Mathematics Graduate Program by January 15 of their final year in the M.S. program.

The letter of application must contain the names of at least three faculty members who may be contacted for letters of recommendation. These three references should include an individual who potentially may serve as the student's doctoral advisor. (The faculty members need not all be associated with UVM.)

The application letter must also contain a preliminary plan and timetable for completing the Ph.D., including a possible research area, a potential thesis advisor, and course work (including reading courses) that might be needed to carry out the plan.

Admission criteria will include: the quality and level of graduate work done at UVM or other institutions, fit of the applicant's research plan within the areas of expertise of the UVM faculty, prognosis for completion of the applicant's plan, consideration of funding for the student (where applicable) during his/her doctoral studies, and comparisons with the pool of external applicants (who are also subject to these criteria). Students accepted to the doctoral program do not need to retake any UVM mathematics qualifying exams they have already passed at the PhD level; however, they must pass both written qualifying exams at the PhD level prior to January of their 2$^{nd}$ year in the PhD program. Ideally, current MS students looking to transition into the PhD program will have passed the qualifying exams at the PhD level during their time in the MS program, but this is not a requirement.

Applications will be reviewed by the Mathematics Graduate Committee and will be informed of their status by no later than April 15 following their application.

## Funding for Graduate Students

The Mathematics Graduate Program currently has 15 Graduate Teaching Assistantships (GTAs), and, on average, about 3 of these become available to new incoming full-time students each year. The stipend for an MS GTA is approximately $18,500, and for a PhD GTA the stipend is roughly $21,000. These assistantships also carry a scholarship covering up to 9 credit hours of graduate credit per semester during the period of the award.

Graduate Teaching Assistantships are usually awarded (and renewed) for an academic year beginning with the fall semester. *Applicants whose files are complete by January 15 will receive full consideration for a GTA beginning the next fall semester. Applications that become complete after January 15 will be evaluated on a continuing basis for GTA support until all assistantships have been awarded.*

Duties of a Teaching Assistant usually involve teaching one section of an elementary mathematics course per semester (normally 3 teaching hours per week) and conducting help sessions (1 to 2 hours per week). Occasionally, one or more GTA positions will become available at the start of the spring semester. In such a case, all unsupported students currently in Mathematics Graduate Program as well as applicants who have indicated they wish to begin that spring will be considered for the award(s). Master's students serving as GTAs in their first year will normally be renewed for a second year based on good performance as a teacher and a student. Doctoral students may be supported on faculty research grants but are also funded by GTA positions to provide them with important teaching experience. The net amount of support from a research grant is usually comparable to that of a GTA.

GTAs are open to all applicants, domestic and international, with preference given to PhD candidates.

**Fees for Graduate Students**

Graduate students are responsible for certain fees (some are *not* covered by a GTA stipend). The major ones are combined in a Comprehensive Fee. Complete information about all fees may be obtained from the Graduate College's web pages.

## Information for New Students

The college runs an orientation for new students during the week before fall classes begin. Students are strongly encouraged to attend, and new recipients of Graduate Teaching Assistantships are required to attend.

There are also departmental orientation activities during that week. During the orientation, you will learn your way around the department and meet the faculty, staff,and other graduate students. You will be given office assignments, computer accounts, and student identification cards. We will help you register for your courses and give you an introduction to the University computing system, including how to use UVM e-mail.

Graduate Teaching Assistants (GTAs) will find out their teaching assignments by the week before classes at the latest, with the goal of scheduling the assignment to accommodate courses the GTAs will enroll in themselves. Course syllabi and teaching methodology will be discussed during orientation. Generally, GTAs teach one course per semester or assistant a specific faculty member. GTAs control their own course, planning and giving lectures, designing, and grading examinations, and assigning the final grade. In some multi-section courses, there are common syllabi and exams, supervised by a course leader. Usually, GTAs teach an entry-level course. Typical topics involve elementary algebra, trigonometry, finite mathematics, and an introduction to calculus. We provide a sample syllabus and a textbook. GTAs must attend our teacher-training sessions during orientation. These sessions include delivering practice lectures and discussing common issues that arise.

We may have one to two classes available each summer for graduate students to teach. There is no guarantee that these classes will have sufficient enrollment to be offered, and cancellations can occur. Department faculty receive first priority for teaching assignments during the summer, followed by Ph.D. students doing research towards their dissertation. We generally do not offer graduate-level classes in the summer.

GTAs should be aware that the first paycheck does not arrive until the middle of September (the last paycheck is at the end of May, after the end of spring semester). Students may request an advance on their salary under extreme circumstances. For more information, please see the department admins, Kiki Reno or Barbara Asiimwe in the Math/Stat office.

## Graduate Student Responsibilities

### *As a Member of the Community*

Graduate students are strongly encouraged to attend the department colloquia and seminars and engage with the community mathematically as much as possible. At seminar and colloquium invited speakersmake presentations on topics of current interest in mathematics, and often provide introductorytalks especially suited for graduate students. Watch for the announcements! These are great opportunities to engage with researchers.

Also, students should be looking to attend conferences in their area. Here are some places you can find conferences and seminars:

https://researchseminars.org
https://www.mathmeetings.net

Conferences often have funding available for graduate students to attend, so if you see something that interests you, you should contact the organizers and politely ask if they have funding for graduate students to attend. Funding for such conferences usually includes funding for airfare and travel (often funding can be partial).

### *As Teachers*

Attend the training sessions offered by the department for first-time GTAs and perform your teaching duties responsibly. In particular, hand out a syllabus the first day of classes detailing

what material will be covered, the exam schedule, and the criteria for grading. A special rule here at UVM: you may not schedule an exam during the last 5 days of classes, so plan ahead. A very helpful listing of standard teaching practices at UVM is available on the department website.

**TAs should not cancel classes** without first making every effort to find a substitute teacher. The Graduate Director should be notified of all substitute teaching and any required class cancellations. TAs need to hold regularly scheduled office hours, a total of at least 3 hours per week, and inform the department of when they are (forms for this purpose are distributed at the beginning of each term). These hours should also be posted on course websites and on office doors.

Graduate Teaching Assistants are required to staff the Help Sessions. This responsibility is just as important as classroom teaching. Any TA who cannot be present must find a substitute. TAs should encourage their students to visit the Help Sessions for help outside of classes.

The graduate director, the course coordinators, and other mentors will be closely monitoring and evaluating TAs' teaching effectiveness. They are available to help TAs if any questions related to teaching should arise.

## Advisor/Advisee Relationships

Students who wish to do thesis work are responsible for finding an advisor. The department is in no way responsible for matching students with advisors nor are students guaranteed an advisor when entering the program. Once a student has an advisor, the student is able to switch advisors (and advisors are able to end the advisor advisee relationship). After termination of an advisor advisee relationship students are subject to the same deadlines for completion of their academic program. Students who have passed the qualifying exam at the level required for their degree may be dismissed from the program if they do not have an advisor for more than two semesters (if an advisor/advisee relationship is terminated, the semester in which the relationship was ended counts as the first semester).

## Code of Conduct

Students are responsible for their behavior at University of Vermont or any time they are representing University of Vermont. This especially includes while they are on campus, at department functions, visiting other institutions, or at conferences. In particular, we expect students not to harass others, not to abuse drugs or alcohol, and to uphold a high standard of academic integrity.

Violations of the Code of Conduct include but are not limited to:

- Any form of harassment. This includes but is not limited to: attacking others (verbally or physically), coercion, persistent disrespectful behavior; any form of sexual harassment; issuing threats; ignoring individuals requests to stop a certain behavior; disorderly and disruptive conduct in classes, seminars, meetings, conferences, or any other professional or university function (for example yelling at another student or professor in an aggressive manner is not acceptable); doxing; spamming emails or abusing other university systems;
- Academic dishonesty. This includes but is not limited to cheating on exams and coursework, including the comprehensive and qualifying exams; plagiarism; misrepresenting yourself or your work.
- Destructive behavior. This includes but is not limited to intentional destruction property of the university, department, or others; removing or stealing furniture, blackboards, or

equipment from offices or other university spaces; graffiti etc.

Students are also subject to the Graduate College code of conduct as well.

## Department and University Awards

There are several awards programs in the department and university for graduate students.

Each year at Honors Day, the department gives the John F. Kenney Award to the outstanding graduate student in mathematics. The department also gives the Sang Kil Nam Scholarship Award to an outstanding undergraduate or graduate student in mathematics and statistics.

The department has one free one-year membership in the Mathematical Association of America and several one-year memberships in the American Mathematical Society. The Director of the Mathematics Graduate Program awards these to the most qualified eligible graduate students. No student will receive the award more than once.

Each year, the Graduate College holds a university-wide competition for the Graduate Teaching Assistant of the Year. The department may nominate up to three students for this award (two in mathematics, one in statistics).

Each summer, the Graduate College offers a number of Graduate Student Summer Research Fellowships. For details, please contact the Graduate College.

Each fall, the university initiates a *Student-Faculty Research Program*. Students (junior/senior undergraduates and first year graduates) may apply for funding of a joint research project with a faculty member. Funding - including a stipend and supplies - is given for either the spring or summer. Application to this program is competitive and requires submission of a proposal for review by a faculty panel. More information, including deadlines, application forms, etc.is available from UVM's Office of Sponsored Programs (on their web site as well).

Each April, the Graduate College and other offices on campus host the *UVM Student Research Conference*. Students engaged in research projects are strongly encouraged to participate.

## Master's and Doctoral Degree Requirements

The following description of our degree requirements is included for your information. The online Graduate Catalogue contains the formal requirements including rules and regulations, e.g., those concerning residency for in-state tuition.

### Doctor of Philosophy in the Mathematical Sciences

The Ph.D. degree requires 75 semester hours in coursework and dissertation research. Students must maintain a 3.0 GPA. Students are required to pass two written qualifying exams and an oral examination based on written proposal; for details see the subsequent section on Doctor of Philosophy Qualifying Exams. Following successful completion of these exams, the student is admitted to candidacy for the degree. The candidate must then write a doctoral dissertation and pass a final oral defense of that dissertation. At least one semester of college teaching experience is required.

### Master of Science Degree Requirements

Each student must complete one of the following options:

a. Twenty-four hours of acceptable graduate credits in advanced mathematics courses; six hours of thesis research culminating in a master's thesis, or

b. Thirty semester hours of acceptable graduate credits in advanced mathematics courses; no thesis required.

Under either option, students must take, or acquire the knowledge of the content in, the courses Math 331 and 333, and must satisfactorily complete at least four 300-level mathematics courses. In both options, students must select a *major concentration* which is either Pure or Applied Mathematics. The concentration shall consist of at least nine approved credit hours in advanced mathematics courses in the respective area, three ofwhich must be at the 300-level; students in option a. may count the six hours of thesis credit towards these nine hours. With approval of the student's advisor, up to six credit hours of courses outside mathematics may be used to fulfill the degree requirements. Students are strongly encouraged to take only 300-level courses whenever possible.

To complete the MS degree, students must maintain a 3.0 cumulative GPA. They must also pass a written comprehensive exam and either pass a second written comprehensive exam or successfully defend a thesis. The exams are offered in January and August: details can be found later in the handbook. This program usually requires two years (full-time) to complete.

**Concurrent Credit for M.S. and Ph.D. Programs**

Up to 30 credit hours of course work for which graduate credit is earned at UVM in a Master's degree program, whether a Master's degree is received or not, may be applied toward a Ph.D. at UVM, provided they are appropriate for the Ph.D. program. A student may thus potentially complete both the M.S. and Ph.D. degrees with a minimum of 75 credit hours of course and thesis work.

Note that an M.S. in Statistics or Biostatistics can be earned with the Ph.D. in mathematical science. Advising for the M.S. would be arranged through Professor Jeff Buzas in close coordination with the student's Ph.D. advisor.

A third course in the fall of the Second Year could be taken in place of, or in addition to, the Third course in the fall of the First Year.

## Core Courses

Graduate students in are (usually) expected to take the "first year sequence". As such, students preparing to enter the graduate program should take the appropriate courses to prepare for these course. For Pure Mathematics these could include two semesters of real analysis, two semesters of abstract algebra, point set topology, and complex analysis.

Pure mathematics first year sequence.

| Fall | Spring |
|---|---|
| Measure Theory (Math 333) | Complex Analysis (Math 331) |
| Abstract Algebra III (Math 395) | Abstract Algebra IV (Math 395) |
| Algebraic Topology (Math 354) | Topics in Combinatorics |

Applied Mathematics first year sequence.

| Fall | Spring |
|---|---|
| Differential Equations (Math 330) | Partial Differential Equations (Math 339( |
| Numerical Analysis (Math 237) | Numerical Analysis (Math 337) |
| Methods of Applied Mathematics | |

# Written Examinations

The nature and timing of the written comprehensive examinations depend on your degree objective and whether you've concentrated on pure or applied mathematics.

**Master's Comprehensive Examinations**

All students in the non-thesis version of the M.S. degree are required to pass 2 written comprehensive examinations. These written exams are given during the week prior to the beginning of the semester in August and January. MS students must pass both exams by January of the student's second year. Two attempts for each exam are allowed.

Any student planning on taking a written examination should contact the Director of the Mathematics Graduate Program at the end of the prior semester. In consultation with the Graduate Committee, the Director will appoint a committee to author the written examination. The exam generally requires three hours.

Students completing the thesis option of the MS will be examined in their major concentration as part of their (required) thesis defense; therefore, they only need to pass a single written comprehensive exam.

The guidelines for the M.S. written examination only require that the student *has knowledge of* the material; it does not require that the student has taken the corresponding courses. Thus, a student who has entered the program with a strong background may not need to take the courses listed above.

Specific examination topics are listed below, as the exams are the same as those offered to PhD students, but with a different passing criteria. All MS students must pass the real analysis portion of the written examination in analysis. The second exam can be in a topic of the student's choosing.

**Doctor of Philosophy Examinations**

Doctoral candidates are required to pass two qualifying exams and a comprehensive exam.

Students will take written qualifying examinations in a total of two areas (in no particular order). Students with a concentration in Pure Mathematics will take two exams chosen from

1. Real & Complex Analysis (241, 242, 331, and 333)
2. Algebra (251, 252 and 395)
3. Combinatorics and Graph Theory (273 and 373)

Students with a concentration in Applied Mathematics will take exams in

1. Numerical Analysis (237 and 337),
2. Differential Equations (330 and 339).

In addition, all students will take a comprehensive exam consisting of a written and oral presentation on a topic chosen in consultation with their research studies committee, whose members will be present for the oral examination. Topics currently offered include

3a. Number Theory/Arithmetic Geometry (255 and 351 and 395)
3b. Combinatorics (273 and 373)
3c. Functional Analysis (335)
3d. Methods of Applied Mathematics (395)
3e. Complex Systems (266 and 300)
3f. Nonlinear PDEs (395)
3g. Mathematical Biology and Ecology (268)

Students interested in taking an exam on a topic not found in the above list are encouraged to contact the Graduate Director regarding other possible exam topics.

These exams are offered twice a year: at the start of the spring semester (the middle of January), and immediately before the academic year begins (the end of August).

Students are encouraged to try their first qualifying exam by the middle of the first year of study.They must pass both qualifying exams at the PhD level by January of their second year of study.

One to two months in advance of each exam date, the mathematics graduate director will send out an email reminder, which will give a deadline for students to inform the director of their intention to take an exam. The Graduate Committee will prepare, administer, and grade the exams. Students can expect to hear the results two weeks after taking the exam.

Separate syllabi are available in each area. The syllabi as well as some previous exams are posted here: https://www.uvm.edu/cems/mathstat/graduate_programs

**Masters Degrees and Advancement to Candidacy for Doctoral Student**

After successful completion of the qualifying exams, a doctoral candidate can petition for awarding of the M.S. degree. A student may be awarded the M.S. degree if the student fails to pass a particular examination at the PhD. level, but is deemed to have done sufficiently well in their coursework and examinations. Upon successful completion of the Comprehensive Exam, the student can advance to candidacy for the Ph.D. degree.

## Advising and Program Requirements

All graduate students should periodically check the Graduate College's web pages for current students (http://www.uvm.edu/~gradcoll) for relevant information. The director of the Mathematics Graduate Program will pair each student with an advisor from among the Graduate Faculty. Students are welcome to change advisors in accord with their interests.

Because of the size of the graduate program and the need to provide funding for students, applicants to the Ph.D. program are generally not admitted without their first identifying a suitable research and dissertation advisor (or advisors). As soon as possible after matriculation, the student should, in consultation with the Graduate Program Director and identified faculty advisor, establish a Studies Committee. It is the responsibility of the Studies Committee to supervise the graduate student's program and to review progress at regular intervals. The Studies Committee consisting of at least three members of the Graduate Faculty is appointed by the Graduate Program Director and approved by the Dean of the Graduate College. The student's academic/dissertation advisor serves as the Chairperson of the Studies Committee. On occasion, it may be appropriate for a professional other than a member of the Graduate Faculty to serve as a member of the Studies Committee, and in this case written approval must be obtained from the Dean of the Graduate College prior to the student's beginning dissertation research.

The Studies Committee must meet with the student at least annually, and more frequent consultation is both highly recommended and encouraged. A brief report from these meetings certifying appropriate progress toward completion of the degree (e.g., timely completion of required Qualifying Exams) and indicating any items of importance (e.g., agreed-upon timelines for completion of thesis work) appropriate for the record should be prepared for the Graduate Program Director. These reports will be forwarded to the Graduate College and placed in the candidate's graduate record.

The graduate students play an important role in the shaping of our curriculum. The graduate committee welcomes suggestions on course offerings. If several students would like to takea particular course, they should contact the Director of the Mathematics Graduate Program or a professor with interests in that area. This is especially important for topics courses in areas where students want to do thesis research.

Most 200 level courses are available for both undergraduate and graduate credit. In these courses students attend common lectures. Usually, however, the graduate students may be given

additional homework assignments and different exams and held to higher grading standards. Please note that credit for Math 230 does not apply towards a degree in Mathematics. Math 230 is considered elementary material which a student should have had as an undergraduate. A student who needs to review this material may sit in on this course without credit or study the material independently. If they do take the course for credit the tuition willbe covered under a Graduate Teaching Assistantship, but the credits earned will *not* count towards the degree.

Master's students choosing the thesis option may sign up for Math 391, Master's Thesis Research. Doctoral students may sign up for Math 491, Doctoral Dissertation Research. All students are recommended to take Math 330, Advanced Ordinary Differential Equations, but this course is not required.

Students and advisors should carefully read the section on qualifying and comprehensive examinations in this handbook for information on the scheduling, topics, and structure of these exams.

Advisors should note that GTA students carry a tuition waiver covering up to 9 credits per semester. A minimum of 6 credits per semester is needed to maintain full-time graduate student status. Credits above the 9-credit remission per semester are charged to the student on a per-credit basis. However, with the approval of the Graduate College and the student's advisor, it may be possible to waive this extra fee, provided that a student takes at most 24 credits per year. Students interested in this possibility should ask the Graduate College.

Depending on the degree option, a student may be required to pass a thesis defense (applicable to all doctoral candidates, and to those Master's candidates writing a thesis) depending on the program of study. A student writing a thesis or dissertation should consult the Thesis and Dissertation information linked to the Graduate College webpage under "Policies and Procedures."

On occasion a graduate student may desire to postpone or suspend studies for a period of time. Students may apply for up to one year's leave of absence, a semester at a time. Application forms are available at the Graduate College.

A student who has finished all coursework but not yet graduated can sign up for Continuous Registration. This option maintains one's full-time student status while minimizing tuition costs. This is done by registering for a semester of GRAD 903 at a cost of $300 per semester.

# Accelerated Master's Program

The Accelerated Master's Program (AMP) in mathematics is designed so that students with strong ability and motivation can complete a bachelor's degree in mathematics, science, or engineering, as well as a master's degree in mathematics at UVM within five years. Interested students should also see the general description of Accelerated Master's Programs on the Graduate College web pages.

The first four years consist of an undergraduate program that includes the core requirements for a minor in mathematics together with other courses that lay a solid mathematical foundation; this portion culminates in a bachelor's degree. During the fifth year, students take courses that complete the requirements for the master's degree in mathematics.

The AMP in mathematics integrates the undergraduate and graduate experiences so that students receive both the breadth and depth they would achieve had they completed the two degrees separately.

**Requirements for Admission**

The Accelerated Master's Program in mathematics is designed for students who are mathematics majors, or who are majors in science or engineering with a minor in mathematics. A student who is enrolled in this AMP may count 6 credits of Mathematics coursework at or above the 200-level for both the undergraduate and the graduate degrees. If interested in the AMP in Mathematics, a student declares this interest in writing to the Director of the Mathematics Graduate Program before taking any courses that he or she would like to count towards both degrees. The student should also be sure to inform the course instructors that he or she wishes to participate in these courses at the graduate level.

Formal application for the Accelerated Master's Program in mathematics is made during the spring pre-registration period of the student's junior year. The student carries out the usual procedure for admission to the M.S. Program in mathematics, including submission of letters of recommendation. Application forms are available online. *The student's admissions essay must specifically address why the student wishes to enter the Accelerated Master's Program*.

Applicants must achieve the following by the end of their junior year:

1. Completion of Math 21, 22, 52, 121, and 124 with an overall GPA of 3.0 or higher,

2. Completion of Math 241 and Math 242 with grades of B+ or better in each, and

3. Completion of a least two additional 200-level mathematics or statistics courses with grades of B+ or better in each.

**Requirements for Advancement to Candidacy in the AMP**

Students who have been admitted to the Accelerated Master's Program in mathematics normally advance to candidacy in this program at the end of their senior year. This marks the end of their undergraduate curriculum and the beginning of their graduate curriculum. The criteria for advancement to candidacy are:

1. Completion of a bachelor's program in mathematics at UVM, or completion of a bachelor's program in science or engineering at UVM with a minor in mathematics;

2. Completion of at least two additional mathematics or statistics courses at the 200-level with grades of B or better in each (these are in addition to Math 241, 242 and the two 200-level courses required for admission to the program); and

3. Completion of a 300-level course in Mathematics with a grade of B or better. This course may not be counted towards the student's undergraduate degree or GPA, and so must be taken as an overload. Note that AMP students will need to know material from the spring course offering of Math 331, Complex Analysis, by the time they take the comprehensive exam in January of their $5^{th}$ year.

Students who have been admitted to the AMP on the completion of their junior year but who later fail to meet the requirements for advancement to candidacy for the M.S. degree will only be permitted to continue towards their M.S. degree after review by the Graduate Program Committee and with the written approval of the Director of the Graduate Program in Mathematics.

A good piece of advice is to not put off taking second semester real analysis or abstract algebra (242 and 252) after taking the first semester (241 and 251). This is similar to the advice that one should not put off taking calculus II after taking calculus I.

**Requirements for Completion of the Master's Degree Portion of the Program**

A total of 30 total credits are required for the Master's degree, of which 6 may be in Mathematics courses at the 200-level or above that were counted towards the Bachelor's degree, and 3 or 4 may come from a 300-level course taken as an overload in the undergraduate years. Fulfilling the M.S. degree requirements then calls for the completing the remaining credit hours, including two other 300-level mathematics courses. The course of study must include either 6 credits of thesis work or 9 credits in an approved major subject featuring a 300-level course. Students must pass the M.S. written comprehensive exams covering analysis and a second option. These exams will also cover the major subject unless the student has chosen the thesis option, in which case there will be a thesis defense. For more details, see the section of the Handbook for Graduate Studies in Mathematics that is devoted to Master of Science Degree Requirements.

**Sample AMP Schedule**

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|---|---|---|
| First Year, Fall | Math 21 (4hrs) & CS 21 & 3 Electives | 16 hours |
| First Year, Spring | Math 22 (4hrs) & Math 52 & 3 Electives | 16 hours |
| Second Year, Fall | Math 121 (4hrs) & (Math 141,151, or 1XX) & 3 electives | 16 hours |
| Second Year, Spring | Math 124 & (Math 1XX or 2XX) & 3 Electives | 15 hours |
| Third Year, Fall | Math 241 & (Math 230 or 2XX) & 3 Electives | 15 hours |
| Third Year, Spring | Math 242 & Math 2XX & 3 Electives | 15 hours |
| Fourth Year, Fall | Math 333 &251 & 3 Electives | 15 hours |
| Fourth Year, Spring | (Math 252 or 2XX) & (Math 331 or 3XX) & 3 Electives | 15 hours |
| Fifth Year, Fall | Math 2XX & 2XX & 3XX | 9 hours |
| Fifth Year, Spring | (2XX or 3XX) & Math 3XX & 3XX | 9 hours |

For more information and a formal list of requirements please contact the Director of theMathematics Graduate Program. The most important part for AMP students in Pure Mathematics is to take complete 241,242,251,252, undergraduate topology 254, and undergraduate complex analysis 247 before their AMP year.

## Course Descriptions

The following informal course descriptions are included for your information. The catalogue contains the formal course descriptions, credit hours, and prerequisites.

**Mathematics Courses**

**Math 230** *Ordinary Differential Equations* Solutions of ordinary differential equations, the Laplace transformations, series solutions of differential equations. *This course is available for graduate credit for students in other areas, but credit for this course will not count towards amathematics graduate degree.*

**Math 235** *Mathematical Models and Analysis* Methods of applied mathematics.

**Math 237** *Introduction to Numerical Analysis* This course develops methods for obtaining numerical solutions to problems too difficult to be solved "in closed form." In addition, the course explores the effect of fixed precision arithmetic when using a computer with these methods. Topics include: methods for solving f(x)=0 and a discussion on efficiency of the computations; methods for solving systems of *n* equations with *n* unknowns, and the effect of having nearly duplicate information (ill-conditioned systems); the problem of generating a function or functions to describe a given set of data (interpolation, splines, least squares); and the problem of estimating the value of definite integrals too difficult to find antiderivatives for.

**Math 240** *Fourier Series and Integral Transforms* The purpose of the course is to study integral operators and related Fourier series expansions which are useful in solving initial and boundary value problems for differential equations. Topics covered include Green's functions for initial and boundary value problems, Laplace transforms, classical and more general Fourier expansions, eigenvalue methods for boundary value problems.

**Math 241** *Real Analysis I* Much of mathematics is traditionally divided into the general areas of Abstract Algebra and Analysis. In our curriculum, these topics are introduced in Math 251 and Math 241. Math 241, Real Analysis I, may be viewed as an extension of the main ideas from Calculus: Limits, derivatives, integrals, and convergence of sequences and series. In this course the student studies the basic properties of sets of real numbers, including the topology of Euclidean *n*-space, and analyzes the behavior of real-valued functions defined on such sets. This sets the stage for generalizations such as abstract metric spaces and other more modern notions.

**Math 242** *Real Analysis II* This is the spring semester continuation of Math 241. Topics normally include: differentiation in $\mathbf{R}^n$, Riemann-Stieltjes integral, uniform convergence of functions, Inverse and Implicit Function Theorems.

**Math 251** *Abstract Algebra I* Basic theory of groups, rings, fields, homomorphisms, and isomorphisms. The course is set at the level of the text *Abstract Algebra, Second Edition* by D. Dummit and R. Foote (John Wiley & Sons, Inc.)

**Math 252** *Abstract Algebra II* This is the spring semester continuation of Math 251. Topics include: modules, vector spaces, linear transformations, rational and Jordan canonical forms, finite fields, field extensions, and Galois Theory leading to the insolvability of quintic equations.

**Math 254** *Point-Set Topology* Topological spaces, closed and open sets, closure operators, separation axioms, continuity, connectedness, compactness, metrization, uniform spaces.

**Math 255** *Number Theory* Number theory has been called the "Queen of Mathematics", because it involves the development of a wide variety of useful methods and produces beautiful and profound results concerning one of the most basic objects in mathematics: the natural numbers. This course allows one to appreciate the work of many mathematical giants and some of the relationships between geometry, algebra, and analysis. Those interested in the history or teaching of mathematics, or in obtaining a concrete introduction to some of the concepts of abstract algebra will find this to be an attractive course which can be taken early in the major.
The ideas found here, originally thought to be purely theoretical, now find applications of critical importance to society.

**Math 257** *Topics in Group Theory* Topics may include abstract group theory, representation theory, classical groups, or Lie groups.

**Math 259** *Cryptography* Topics include specific public-key and private-key cryptosystems such as RSA, ElGamal, and elliptic curve cryptosystems, as well as digital signatures and key exchange. Related mathematical topics of congruences, factoring, finite fields, primality testing, probability, and the arithmetic of elliptic curves.

**Math 266** *Chaos, Fractals, and Dynamical Systems* Discrete and continuous dynamical systems, Julia sets, the Mandelbrot set, period doubling, renormalization, Henon map, phase plane analysis, and Lorenz equations.

**Math 268** *Mathematical Biology and Ecology* Mathematical modeling in the life sciences. Topics include population modeling, dynamics of infectious diseases, reaction kinetics, wave phenomenon in biology, and biological pattern formation.

**Math 272** *Applied Analysis* Fourier series; solution of the wave, heat, and Laplace equations by eigenfunction expansions; Fourier-Bessel series; complex variables; conformal mapping; Schwarz-Christoffel mappings.

**Math 273** *Combinatorial Graph Theory* A graph is a collection of points together with lines joining pairs of these points. Graphs are used to model a wide variety of situations. For example, the points may represent microprocessors with the lines representing communication channels. When can we lay out the computer circuit without crossing wires? Or perhaps the points of a graph represent people and the lines indicate friendships. When can we pair together the people so that everyone is with a friend? Or perhaps the points are airports and the lines are flights. How can a traveler arrange his or her schedule to minimize the total cost of her trip? These and other topics are explored in Math 273, Combinatorial Graph Theory. The questions above are respectively special cases of graph planarity, matching theory, and the Hamilton cycle problem. You will also study paths and trees in graphs (connection problems), Eulerian cycles (postman problems), and point and line colorings (storage and scheduling). Finally, what does all this have to do with the five Platonic solids and the thirteen semiregular polyhedra? Take the course and find out!

**Math 274** *Numerical Linear Algebra* The purpose of the course is to study numerical algorithms for solving systems of linear equations, manipulating matrices, solving least squares problems, determining rank computationally, finding eigenvalues, and determining sensitivity of solutions to perturbations. The course emphasizes the importance of determining numerical stability of algorithms since floating point computation inevitably introduces errors compared with ideal exact computations.

**Math 300** *Principles of Complex Systems* Introduction to fundamental concepts of complex systems. Topics include: emergence, scaling phenomena, and mechanisms, multi-scale systems, failure, robustness, collective social phenomena, complex networks. Students from all disciplines welcomed. Pre/co-requisites: Calculus and statistics required. Linear Algebra, Differential Equations, and Computer programming recommended but not required. Cross-listing: CSYS 300. Credits: 3

**Math 303** *Complex Networks* Detailed exploration of distribution, transportation, small-world, scale-free, social, biological, organizational networks; generative mechanisms; measurement and statistics of network properties; network dynamics; contagion processes. Students from all disciplines welcomed. Pre/co-requisites: Math 300/CSYS 300, Calculus, and Statistics required. Cross-listing: CSYS 303. Credits: 3

**Math 330** *Advanced Ordinary Differential Equations* Linear and nonlinear systems, approximate solutions, existence, uniqueness, dependence on initial conditions, stability, asymptotic behavior, singularities, self-adjoint problems.

**Math 331** *Theory of Functions of Complex Variables* Differentiation, integration, Cauchy-Riemann equations, infinite series, properties of analytic continuation, Laurent series, calculus of residues, contour integration, meromorphic functions, conformal mappings, Riemann surfaces.

**Math 333** *Theory of Functions of Real Variables* The theory of Lebesgue integration, Lebesgue measure, sequences of functions, absolute continuity, properties of $L^p$ -spaces.

**Math 335***Advanced Real Analysis* $L^2$-spaces, $L^p$-spaces; Hilbert, Banach spaces; linear functionals, linear operators; completely continuous operators (including symmetric); Fredholm alternative; Hilbert-Schmidt theory; unitary operators; Bochner's Theorem; Fourier-Plancherel, Watson transforms.

**Math 337** *Numerical Differential Equations* Numerical solution and analysis of differential equations: initial-value and boundary-value problems; finite difference and finite element methods. Prerequisites: MATH 237, either MATH 230 or 271 recommended. Credits: 3

**Math 339** *Partial Differential Equations* Classification of equations, linear equations, first order equations, second order elliptic, parabolic, and hyperbolic equations, uniqueness and existence of solutions.

**Math 351** *Topics in Algebra* Topics include algebraic number theory, commutative algebra, modular forms, representation theory, algebraic geometry, and the arithmetic of elliptic curves. May be repeated for credit with the permission of the instructor and the advisor.

**Math 354** *Algebraic Topology* Homotopy, covering spaces, homology and cohomology theories, fixed point theorems.

**Math 373** *Topics in Combinatorics* Topics will vary each semester and may include combinatorial designs, coding theory, topological graph theory, cryptography.

**Math 391** *Master's Thesis Research* Provides up to 6 credits for research and preparation of a M.S. Thesis.

**Math 395** *Special Topics* Courses which are relatively new or offered infrequently are assigned this number. There can be various sections covering various topics. Recently this has included Algebra III, Algebra IV, Probability Theory, and Topics in Analysis.

**Math 491** *Dissertation Research* This course provides credit for the research and preparation of your Ph.D. dissertation.

**Statistics Courses**

**Stat 200** *Medical Biostatistics* Introductory design and analysis of medical studies. Epidemiological concepts, case-control, and cohort studies, and clinical trials. Students evaluate statistical aspects of published health science studies. (Required for Biostat M.S.)

**Stat 211** *Statistical Methods I* Fundamental concepts and techniques for data analysis and experimental design, descriptive and inferential statistics, including classical and nonparametric methods, regression, correlation and analysis of variance. This course is taken as an introduction by graduate students of various disciplines (but would not count as credit for the statistics or biostatistics M.S. degrees).

**Stat 221** *Statistical Methods II* Multiple regression and correlation, basic experimental design. analysis of variance (fixed, random and mixed models) and analysis of covariance. Computer software usage. (Required for the stat/biostat M.S.)

**Stat 223** *Applied Multivariate Analysis* Analysis methods for categorical and continuous multivariate data, discriminate analysis, logistic regression, canonical correlation, principal components, factor analysis and log linear models, computer software usage. (Required for the stat/biostat M.S.)

**Stat 224** *Statistics for Quality and Productivity* Statistical methods for product quality and productivity, statistical process controls Shewhart and CUSUM, contorl charts, acceptance, continuous, sequential sampling. Selected statistical computer programs utilized. (Required for the stat M.S.)

**Stat 225** *Applied Regression Analysis* Regression analysis is concerned with how to predict the values of one variable ($y$) from the given values of other variables ($x_1$, $x_2$, …); the simplest example being a straight line (linear) relation $f(x)=ax+b$. With real data collected on $y$ and $x$ the function $f$ never exactly explains $y$ in terms of the $x$values, and so one typically uses a least squares error approach to obtain the best fitting functions to the data. Regression analysis is the basic method used in business, economics and other social/behavioral sciences, health and other biological sciences, and engineering and other physical sciences to fit mathematical models of real data. The standard models and assumptions are reviewed, fitting methods discussed (with software and appropriate matrix methods), and methods of checking the fit (residual analysis) are covered. Linear, multiple, polynomial, logistic, and ridge regressions are included. Important forecasting applications provide some of the examples.

**Stat 231** *Experimental Design* Randomization, complete and incomplete blocks, cross-overs, covariance analysis, factorial experiments, confounding, fractional-replication, nesting, split-plots, repeated measures, response surface optimization, Taguchi methods, and optimal designs. (Required for the stat/biostat M.S.)

**Stat 233** *Design of Sample Surveys* Design and data analysis for sample surveys. Simple random, stratified, systematic, cluster, multistage sampling. Practical issues in planning and conducting surveys.

**Stat 237** *Nonparametric Statistical Methods* Nonparametric and distribution free methods; categorical, ordinal and quantitative data; confidence intervals; rank and Chi-Square hypothesis tests; computer-intensive procedures (Bootstrap, exact tests).

**Stat 241** *Statistical Inference* Introduction to statistical theory: related probability fundamentals, derivation of statistical principles and methodology for parameter estimation and hypothesis testing. (Required for biostat M.S.; but mathematics graduates would be recommended to take Stat 261.)

**Stat 251** *Probability Theory* Distributions of random variables and functions of random variables. Expectations, stochastic independence, sampling and limiting distributions (central limit theorems). Concepts of random number generation. (Required for stat M.S.)

**Stat 253** *Applied Time Series and Forecasting* A fundamental problem in business, economics, engineering and other sciences is providing forecasts for the future values of data collected in time order. Modern statistical models (autoregressive integrated moving average models) are developed in this course, which can be used in the analysis of time series data, fitting models to these data, checking forecast errors, and providing future forecasts. Seasonal effects are considered as well. If information from other series ($x$) can be used to aid in the forecast of $y$, then one can use transfer function models for the analysis. You gain practical experience in fitting a variety of real series with the latest software methods.

**Stat 261** *Statistical Theory I* Point and interval estimation, hypothesis testing, and decision theory. Applications to areas such as nonparametric tests, sequential analysis and linear models. (Required for stat M.S.)

**Stat 287** *Data Science I* Data harvesting, cleaning, and summarizing. Working with non-traditional, non-numeric data (social network, natural language textual data, etc.). Scientific visualization using static and interactive "infographics". A practical focus on real datasets, and developing good habits for rigorous and reproducible computational science.

**Stat 330** *Bayesian Statistics* Introduction to Bayesian inference. Posterior inference, predictive distributions, prior distribution selection. MCMC algorithms. Hierarchical models. Model checking and selection. Use of computer software.

**Stat 355** *Statistical Pattern Recognition*
Analysis of algorithms used for feature selection, density estimation, and pattern classification, including Bayes classifiers, maximum likelihood, nearest neighbors, kernels, discriminants, neural networks and clustering.

**Stat 360** *Linear Models*
Theory of linear models, least squares and maximum likelihood estimation, fixed, random and mixed models, variance component estimation, introduction to generalized linear models, bootstrapping.

**Stat 387** *Data Science II* Advanced data analysis, collection, and filtering. Statistical modeling, monte carlo statistical methods, and in particular Bayesian data analysis, including necessary probabilistic background material. A practical focus on real datasets and developing good habits for rigorous and reproducible computational science.