MODELLING OF THE TRANSPORT OF GLACIAL DETRITUS IN GREENLAND: APPLICATIONS OF COSMOCHRONOLOGY

A Thesis Proposal Presented

by

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Introduction

Over recent decades, it has become apparent that human production of CO_2 and other greenhouse gasses is increasing global temperature on earth, and is likely to continue to do so [Solomon et al. 2007]. However, the precise magnitude of this coming increase is still poorly constrained, in large part because there is no paleoclimatic analogy for a sudden doubling of atmospheric carbon [Hansen 2005]. Of the adverse effects of global warming, the melting of continental ice sheets and the consequent sea level rise have the potential to wreak significant havoc with human civilization [Solomon et al. 2007]. Consequently, an understanding of the sensitivity of continental ice sheets to temperature changes will allow improved projections of coming environmental changes and therefore enable humanity to better prepare for them.

The rate at which ice sheets respond to temperature shifts has been a topic of considerable debate, with new evidence of fast flow regimes (such as surging outlet glaciers) challenging the traditional conception that significant change in mass balance would occur over thousands of years [e.g. Alley et al. 2005]. Less attention has been paid to the overall magnitude of expected glacial mass balance changes, as this depends on the magnitude of changes in atmospheric greenhouse gasses and the sensitivity of global temperature to those changes, as well as the sensitivity of continental ice masses to temperature and attendant precipitation variation. Most research along these lines is based entirely on mathematical models which are ultimately grounded in our understanding and parameterization of both ice physics and present ice sheet conditions [Huybrechts 2006]. Such mathematical models can use the ice core paleoclimate record to reconstruct past ice sheet conditions [e.g. Lhomme et al. 2005] or use temperature projections to model possible future climate scenarios [e.g. Alley et al. 2005].

Various workers [e.g. Håkansson et al. 2007] are attempting to reconcile model results with moraine and cosmogenically-derived age estimates in Greenland and Antarctica to assess the extent of the ice sheets during the last glacial maximum. However, there have been few empirical studies directly examining ice extent during interglacial periods. As ice sheet behavior during warm periods is most crucial for predicting change in ice sheet mass balance from anthropogenic warming, it is timely and important that we attempt to gather empirical data to calibrate modelled ice sheet extent for the interglacial periods.

During interglacial periods, when ice sheet extent was minimal, interior bedrock surfaces and tills would have accumulated cosmogenic isotopes during surficial exposure [Nishiizumi et al. 1996]. During subsequent glacial advances, clasts incorporated into the basal debris of the ice sheet can retain a record of their past exposure history in the form of cosmogenic isotope inventories. We therefore propose to go to the Greenland Ice Sheet margin and sample clasts incorporated in basal ice that are melting out of the ice sheet. We will then analyze the cosmogenic isotope concentrations in these clasts and use these data to reconstruct the exposure and burial histories of the sampled clasts. Ashley Corbett will develop the cosmogenic isotope data analysis in her thesis. Using Corbett's data as boundary condition control, I propose to model basal sliding and the transport of basal debris in the Greenland Ice Sheet from the period of initial exposure until the present. From these model results, I expect to estimate a range of possible source locations for the clasts, thereby providing a minimum extent of the Greenland Ice Sheet retreat during the warm periods indicated by the clast exposure ages.

The first objective of this study is to determine the age and duration of past surficial exposures experienced by the detrital clasts collected from the Greenland Ice Sheet's western margin. Our second objective is to model the transport of these detrital clasts to the Greenland Ice Sheet margin, thereby constraining the geometry of the Greenland Ice Sheet during the periods of surficial exposure. Our last objective is to use these models to inform the discussion of Greenland Ice Sheet extent during interglacial periods, possibly ruling out or confirming models discussed in the literature. These endeavors will inform the larger scientific discussion about ice sheet response to past warm periods and will help scientists predict ice sheet response to future warm periods.

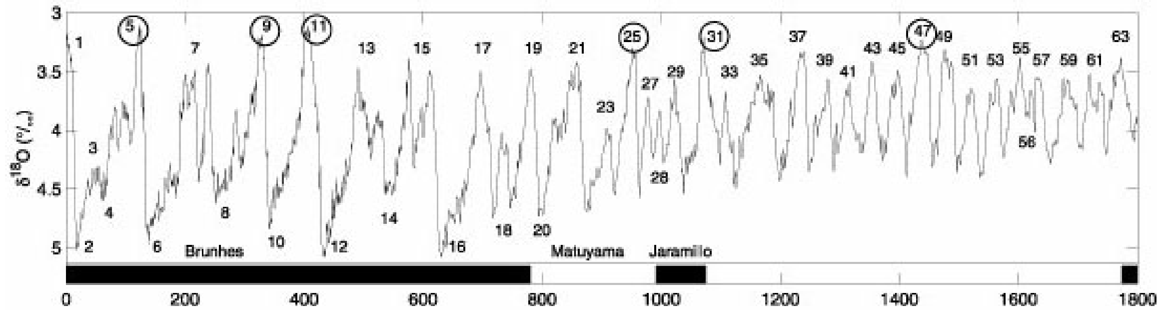

Figure 1. $\delta^{18}O$ record reconstructed from 57 benthic proxy records (from Lisiecki and Raymo, 2005). Oxygen Isotope Stages with values at or above present levels are circled. At these times significant ice sheet retreat is likely.

Background

Paleoclimate Record

The marine oxygen isotope record indicates six periods during the past two million years when the global temperature was at or exceeded present levels [Lisiecki and Raymo 2005]. As our methods of cosmogenic burial dating (described below) may allow for the detection of burial ages up to 2 million years before present [Granger and Muzikar 2001], we may be able to find evidence for ice sheet retreat from any of the warm periods within this 2 million year range. These periods are oxygen isotope stages (OIS) 5, 9, 11, 25, 31, and 47 (figure 1).

Biologic paleoclimate evidence robustly demonstrates substantial northern hemisphere warming during OIS-5, which corresponds to the Eemian period approximately 130 ky, with pollen in Northern Europe showing plant assemblages indicative of conditions several degrees warmer than present [Aalbersberg and Litt 1998]. In southern Greenland, plant macrofossils also show biotic assemblages indicative of 5 degrees warmer than today [Bennike and Bocher 1994].

Less evidence has been collected for earlier interglacial periods. Cosmogenic $^{10}Be/^{26}Al$ data taken from a rock core beneath the GISP2 ice core shows burial ages of 0.5 ± 0.2 million years [Nishiizumi et al. 1996]. As the GISP2 core was taken at Summit Station on the ice divide in central Greenland, this would indicate a total or near-total melt of the Greenland Ice Sheet consistent with OIS-11 approximately 400 ka. However, a shifting ice divide may allow for some portion of the Greenland Ice Sheet to have remained [c.f. Marshall and Cuffey 2000].

Modelling of Past Interglacial Periods

Modelling efforts have focused on OIS-5 ice sheet geometry reconstruction; as this is the last interglacial period, it is easiest to reconstruct from present conditions and ice core records. The earliest efforts at modelling this period show some reduction, though not complete melting, in Greenland's southern dome, consistent with 1-2 meters of sea level increase [Letreguilly et al. 1991]. As sedimentary records indicate an Eemian sea-level highstand of at least 5-6 meters [Vezina et al. 1999], this result implies that substantial melting would have had to occur in the southern hemisphere to make up the difference. These early models were working with only the Dye-3 and Camp Century Greenland ice core records and made the crucial error of assuming that $\delta^{18}O$ variation in the ice core corresponds with temperature variation analogously with present conditions. However, Cuffey et al. [1995] used temperature measurements from the GISP2 borehole to calibrate the relationship between $\delta^{18}O$ and temperature through the last glacial cycle. Once these corrections are made, models show a much-reduced Greenland Ice Sheet, lacking most to all of the southern dome and substantial retreat from the northern margin [Cuffey and Marshall 2000] (figure 2), contributing as much as 5.5 meters to sea level rise [Huybrechts 2002].

However, substantial controversies still exist about how to best model the Greenland Ice Sheet during the Eemian period. Cuffey et al.'s borehole temperature method does not decisively determine the $\delta^{18}O$ / temperature sensitivity for the Eemian period, as geothermal heat flux has affected borehole temperatures in Eemian ice [Cuffey and Clow 1997; 1995]. Most models assume a uniform geothermal gradient beneath Greenland, which is certainly not the case [Braun et al. 2007; Greve 2005]. Modelling efforts that include geothermal gradient variability result in a somewhat larger Eemian ice sheet [Tarasov and Peltier 2003]. The pressure of the gas trapped in Eemian ice indicates substantial ice thickness at Summit during Eemian time [Raynaud et al. 1997]. This along with other ice stratigraphic considerations from the NGRIP and Camp Century cores led Lhomme et al. [2005] to model only minor retreat from Greenland's northern margin during Eemian time, though they still modelled a complete melt of the southern dome. Given the uncertainties in Cuffey et al.'s temperature reconstruction methods, Otto-Bliesner et al. [2006] instead base their temperature reconstructions (as well as precipitation-driven mass balance changes) on the Community Climate System Model (CCSM). Their ice model shows little retreat in the main dome of Greenland, and significant but not complete melting of the southern dome, producing only 3.4 m of sea level rise. The CCSM Greenland temperature projections for the Eemian period are similar to those forecasted for 2100 A.D., given current greenhouse gas production [Overpeck et al. 2006].

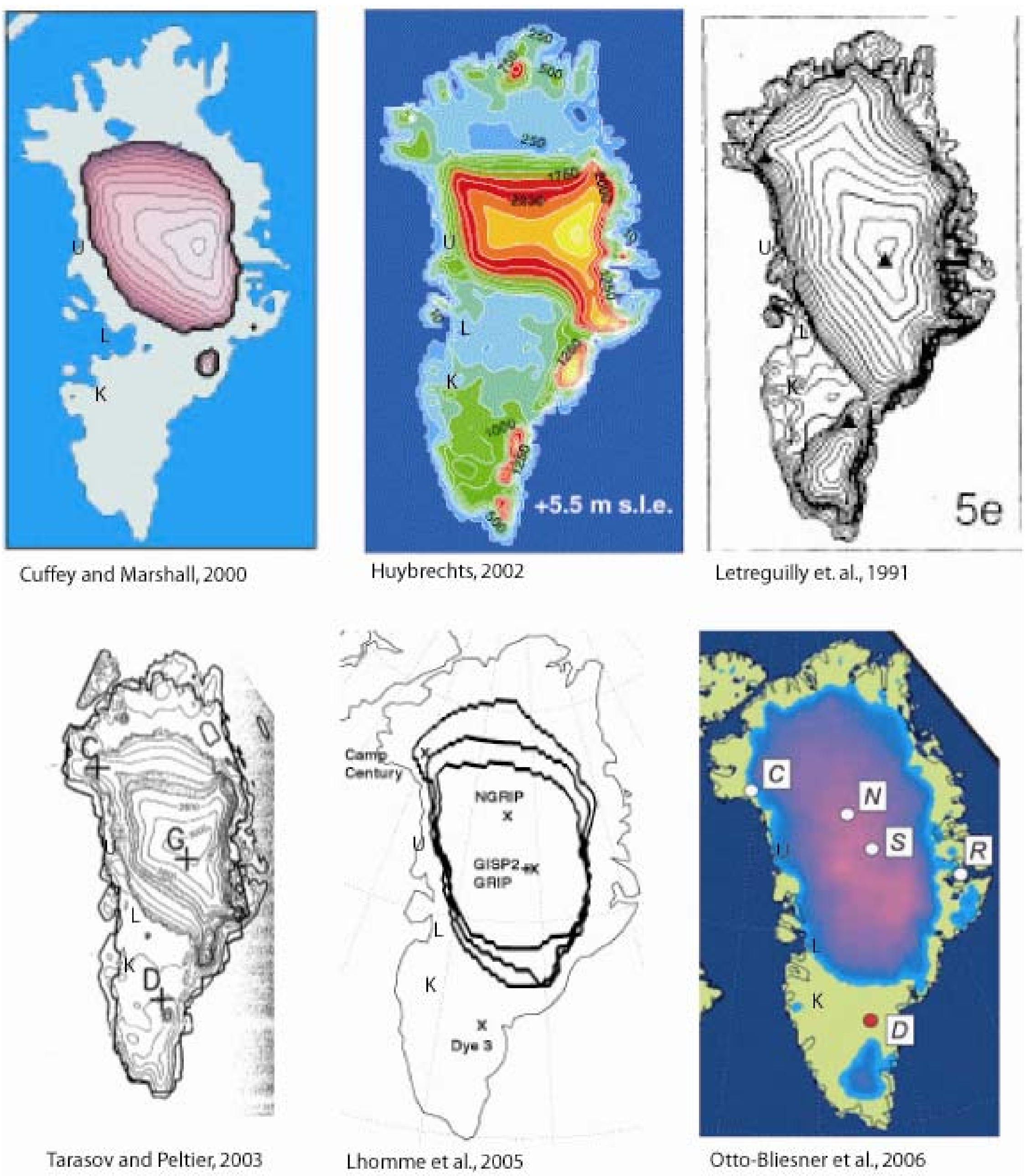

Figure 2. Six different models for Eemian (~130k-124k) extent of the Greenland Ice Sheet. Approximate locations of the proposed study sites are shown: Kangerlussuaq (K), Ilulissat (L), and Upernavik (U)

Cosmogenic dating

Cosmogenic isotopes are generated by the flux of cosmic rays from the galaxy, which collide with surficial materials and split atoms within the crystalline structure of rocks [Lal and Peters 1967]. For example, ^{10}Be is produced by the spallation (splitting) of Si, O, Mg, or Fe into ^{10}Be and ^{7}Li [e.g. Bierman 1994]. Most cosmic rays in the upper atmosphere are protons; but with increasing atmospheric depth, the cosmic ray flux becomes increasingly dominated by neutrons through interactions of the protons with atmospheric gases. These interactions, as well as the primary cosmic-ray flux, also provide a stream of muons amounting to at most several percent of cosmogenic atomic spallation; this fraction is not insignificant as muon-induced spallation can occur at greater depths than neutron-induced spallation [Lal 1988]. Due to the shielding effects of the earth's magnetic field and the atmosphere, nuclide production rates vary by latitude and altitude, however above the latitude of 45° the variation due to latitude is not significant [Lal 1991].

The basic method of cosmogenic isotope burial dating, for which the theory was laid out by Lal [1991], is the comparison of two cosmogenic isotopes of differing half-lives, typically ^{10}Be and ^{26}Al. These ratios allow determination of both exposure duration and burial duration for a given clast [c.f. Bierman et al. 1999; Granger et al. 1997] (figure 3), as compared to single isotope techniques which are constrained by the assumption of a single exposure period.

The cosmogenic burial method has two important uncertainties. First, there is a shielding effect where the production rate of cosmogenic isotopes from neutron exposure decreases exponentially with depth from the surface, according to the density of that material [Lal 1988]. For example, a clast whose material was 1 meter below a bedrock surface during exposure would have ~17% of the cosmogenic isotope production rate of a clast located at the surface. A clast 2 meters below the surface would have ~2%. As exposure durations are not expected to be greater than 5-10 ka, given Greenland's history of interglacial periods [Lisiecki and Raymo 2005], burial age calculations are not likely to be affected by this uncertainty. However, a suite of clasts with similar calculated burial ages may have diverse calculated exposure ages, if the depth of the clasts from the surfaced varied during exposure.

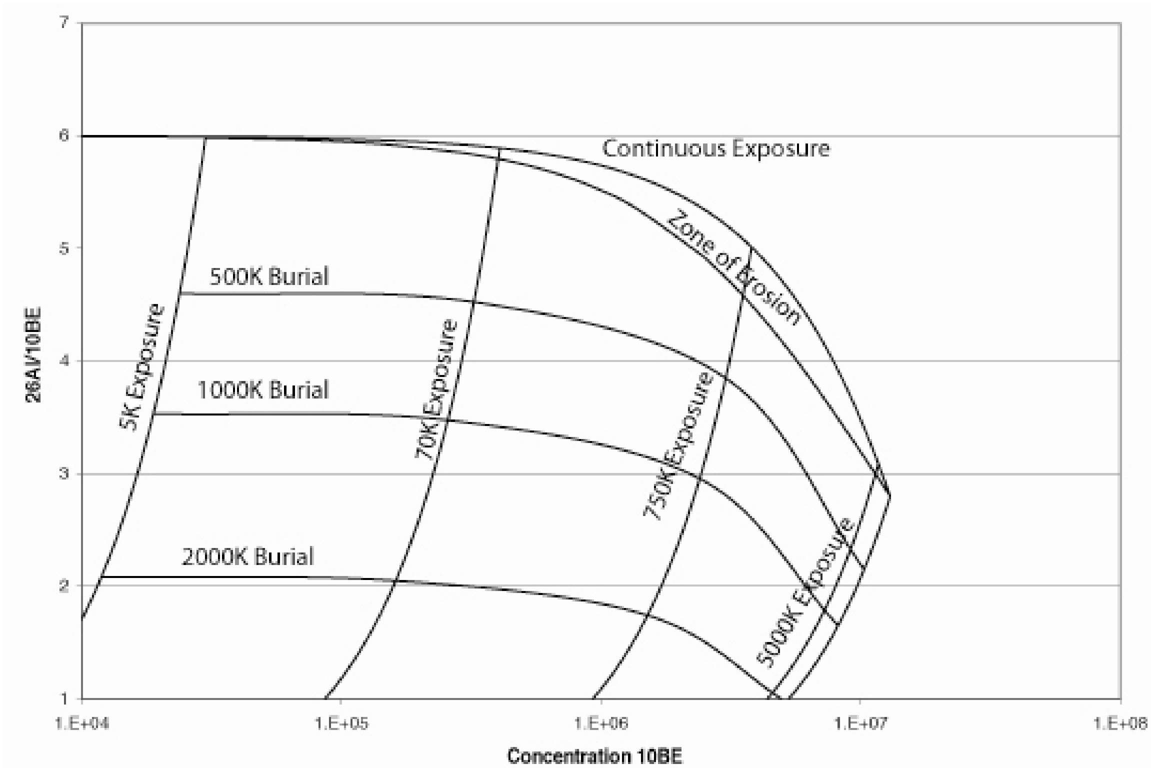

Figure 3. Two-isotope diagram for comparing ratios of cosmogenic isotopes [generated using equations from Lal 1991]. The top line shows how ratios develop with continuous exposure, the second line plots erosion at steady state, creating a zone where eroding surfaces will plot. Exposure and burial isochrons are illustrated.

The second uncertainty is that in a single ^{10}Be /^{26}Al analysis, a complex history of multiple burials and exposures is indistinguishable from a single exposure and burial with the same isotopic outcome. To address this uncertainty, we can examine more than one isotopic ratio. ^{10}Be, ^{26}Al, ^{36}Cl, and ^{14}C have half lives of 1.36 my, 0.7 my, 0.3 my, and 0.005 my, respectively [Bierman 1994; Nishiizumi et al. 2007]. Thus, for example, a clast exposed during OIS-11 400 ka, the Eemian interglacial 127 ka, and the Holocene thermal maximum 8 ka would have more ^{36}Cl compared to ^{14}C than could be accounted for by a single exposure, because ^{36}Cl would remain from its Eemian exposure but ^{14}C would only remain in detectable quantities from its Holocene exposure. Likewise, such a clast would have high quantities of ^{26}Al compared to ^{36}Cl, because significantly more ^{26}Al would remain from the OIS-11 exposure than ^{36}Cl would. Through such multiple isotope comparisons, complex histories of burial and exposure could be detected.

Ice Flow Modelling

Study into the physics of glaciers began in the 1950's. Glen's [1955] flow law establishes a relationship between shear strain rate and shear stress in glaciers, dependent on ice temperature, crystal orientation, impurity content, and other factors [Paterson 1994]. Basic stress and strain equations were derived by Nye [1957], operating under the assumption that ice is homogenous, isotropic and incompressible. These basic equations were demonstrated experimentally and constants determined by various workers throughout the sixties and seventies, both through laboratory experiments and studies of alpine glaciers [Paterson 1994]. Most large-scale ice models employ Hutter's [1983] *Shallow Ice Approximation*, which simplifies the equations by assuming that ice and bedrock slopes are sufficiently small that normal stress components can be neglected [Rutt et al. 2006]. With advances in computer technology, modelling has become increasingly sophisticated, both in the mathematics it employs and in the number of data nodes used in simulating details of glacial phenomena, thus producing increasingly detailed projections of past and future ice sheet conditions [Huybrechts 2006].

As this project proposes to collect basal detritus and use it to further constrain past ice sheet margin geometry, the physics that govern basal flow are essential. The two mechanisms that govern most basal flow are glacial sliding, where a layer of water forms in between the glacier and its bed, and deformation of subglacial till [Paterson 1994]. The first requires that the glacier not be frozen to its bed, though negligible sliding can occur at sub-zero temperatures [Paterson 1994]. The equations that govern glacial sliding were first developed by Weertman [1957]. These assume that the shear stress in the water in between the ice and the bed is negligible and resistance is generated by bumps in the bedrock plane. However, most glaciers are debris-laden at the bed, and debris-bedrock friction generates most of the basal drag. Thus the modelling equations for sliding in debris laden ice relate basal sheer stress to ice velocity, rock on rock friction, debris concentration, and ice viscosity, as well as the roughness characteristics of the bed [Shoemaker 1988].

The present basal conditions under the Greenland Ice Sheet are not well understood, in large part because there have only been four direct measurements of basal temperature, at Camp Century, Summit Station, and the North-GRIP and Dye-3 ice cores

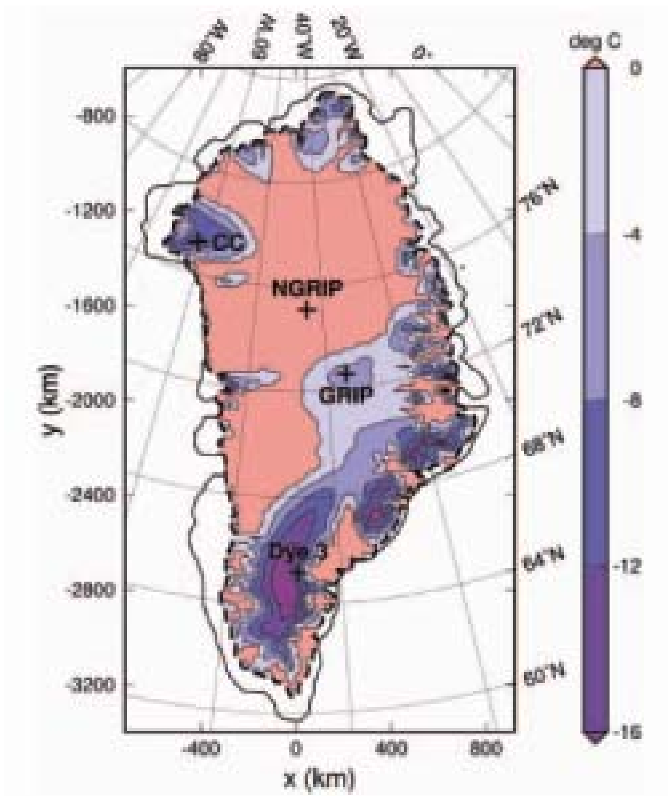

Figure 4. Greve's (2005) map of a model of basal conditions in Greenland. Pink shows above zero basal temperatures and therefore melting. Locations of Dye 3, GRIP, North GRIP, and Camp Century Ice Cores are shown.

[Greve 2005]. Early modelling work by Huybrechts [1996] suggested that most of Greenland was frozen to the bed with pressure melting only occurring in coastal regions and in the central western and northeastern parts of the ice sheet. However, especially since the North-GRIP core reached bedrock in 2003 and basal melting was observed at that location, recent models have shown extensive melting through much of north-central Greenland; though, due to the low temperature recorded at Dye-3, the southern dome is still thought to be largely frozen to the bed [Greve 2005] (figure 4). Perhaps in part because basal processes occur over much of the continent, Greenland outlet glaciers are typically debris rich, containing bands of sand to cobble sized-clasts often several meters thick [Knight et al. 2002; Sugden et al. 1987]. These layers likely form through a process of regelation and sediment accretion, where the excess pressure on the upstream side of bumps in the bedrock surface causes the ice to melt at a lower temperature and refreeze when the meltwater flows around the bump to where the pressure is lower [Paterson 1994]. This melting and refreezing mechanically breaks apart the bedrock, causing the debris to become entrained in the frozen ice. This debris-rich ice both constitutes the sample source for our field collection of clasts and is a boundary condition that we want to include in constructing models of Greenland's basal conditions.

For the modelling component of the project, we intend to use the GLIMMER shallow ice approximation model, developed by Payne [Payne and Dongelmans 1997] and first applied to the modelling of ice streams in Antarctica. GLIMMER employs a set of equations that describe ice thickness, horizontal ice velocity, basal shear conditions, thermal conditions, and isostatic adjustments [Payne and Dongelmans 1997]. The equations for basal heat flux incorporate both factors for friction generated heat and geothermal heat flux [Rutt et al. 2006]. The model then employs a Taylor series function to distribute basal and surface heat throughout the glacier. The sliding velocity is derived directly from basal shear stress, with the ice temperature and the ice sheet geometry providing the ice motion due to creep according to Glen's Flow Law. Ice thickness is adjusted through time according to changes in flow velocity and changes in surface mass balance (i.e. accumulation and ablation). GLIMMER also has the capacity to elastically adjust the lithosphere to the changing ice volume, modelling isostatic rebound or subsidence. The MER in GLIMMER stands for "multiple enabled regions", meaning that the model can model different regions of a larger ice sheet, continuously varying the inputs across the regions, thus producing results over the entire continent and accepting inputs from climate models that vary over a large area.

See Appendix A for a discussion of the physical equations used in GLIMMER.

Study Site

We intend to sample glacial detritus in at least three locations on the ice margin of western Greenland. We will remove clasts either directly from the glacial ice or from detritus known to be recently exposed. We will use the towns of Kangerlussuaq (located at 67° N), Ilulissat (69° N), and Upernavik (73° N) as base locations, accessing the ice margin by foot or by helicopter (figure 5). At Ilulissat we may sample in two locations depending on logistical constraints. The Jakobshaven outlet glacier, near Ilulissat, is one of the fastest in the world [Joughin et al. 2004], and the ice margin near the outlet should have a more dynamic history, perhaps processing material from further in the interior of the continent. From Ilulissat, we may also sample a less dynamic portion of ice margin further north of the outlet glacier.

The locations of our three proposed field sites in Western Greenland are designed to test Eemian ice model results. At 67° N, all of the ice models discussed above show significant or complete melting of ice sheet in Western Greenland during the last interglacial (figure 2). At 73° N, none of the models show significant retreat during the interglacial, but earlier exposures (such as the 11th OIS) may be detected. If Eemian exposures are detected, this would provide empirical data that might necessitate revision of the models. At 69° N, some models show total or near total melt in Western Greenland [e.g. Cuffey and Marshall 2000], while other models show very little melt in this location [e.g. Otto-Bliesner et al. 2006]. Thus, data from Ilulissat, the middle field site, may help resolve some of this controversy.

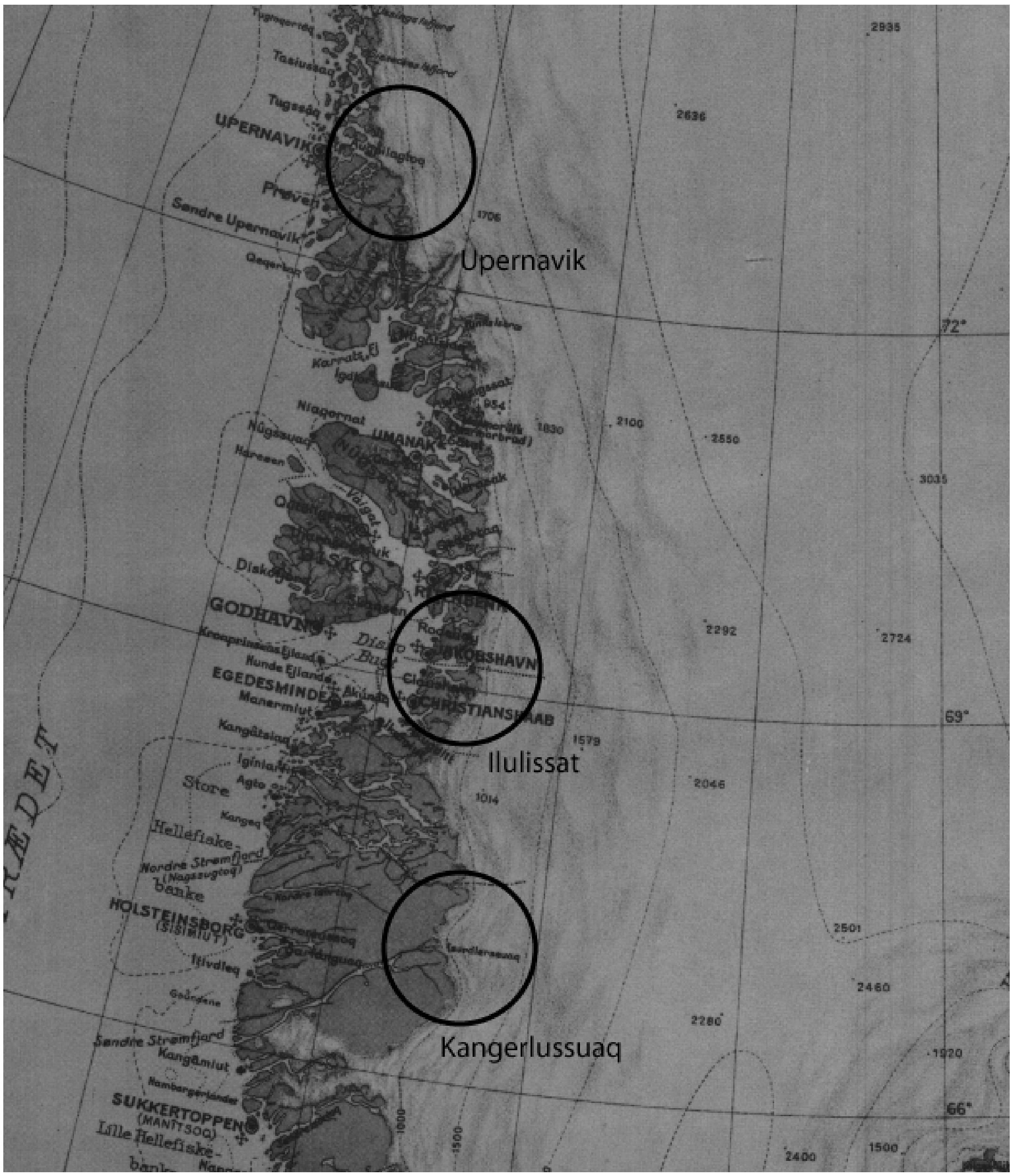

Figure 5. Map of west central Greenland [Geodetic Institute 1938] showing the general locations of our three study sites at Kangerlussuaq, Ilulissat, and Upernavik.

Methods

Cosmogenic Isotope Measurements

We intend to measure the cosmogenic isotope ratios in at least one hundred samples from these locations. We plan to analyze ratios of multiple cosmogenic isotopes, specifically ^{10}Be, ^{26}Al, ^{36}Cl, and ^{14}C, allowing for detection of complex exposure and burial histories through the comparison of younger and older isotope ratios. We may also measure ^{21}Ne in some samples. As ^{21}Ne is both stable and cosmogenic [Bierman 1994], it would allow us to know the total length of exposure for a measured clast. Given the known history of interglacials during the past 2 million years [Lisiecki and Raymo 2005], short exposure histories and long burial histories are likely. Therefore, relatively large samples may be needed to assure that the isotopes are present in detectable quantities. To assure samples contain sufficient Be and Al, we intend to produce around 100 grams of quartz from each sample. ^{36}Chlorine is produced in potassium and calcium-bearing minerals [Stone et al. 1996], so feldspar or amphibole (or possibly mica) will likewise be isolated from samples. 50 grams of potassium or calcium-bearing mineral isolates will be produced from each sample. We will want to have additional sample material available for ^{14}C and ^{21}N analyses, to have the option of making thin-sections, and to have some material left over for unforeseen contingencies. Therefore, we will collect felsic samples containing ~25% quartz weighing more than 1 kg and ideally weighing around 2 kg.

Additionally, we intend to sample the bedrock for cosmogenic isotopes along transects orthogonal to the ice margin at Ilulissat and Upernavik. We hope to learn when the ice sheet retreated in those areas during the Holocene (though older exposures may be detectable). This information will provide additional boundary condition controls for the modelling component of the project, allowing us to more accurately describe the ice margin behavior during the recent Holocene. Such work has already been done in the vicinity of Kangerlussuaq [Rinterknecht et al. 2005].

Modelling

GLIMMER is publicly available, and Jesse Johnson of the University of Montana currently heads a NSF-funded project to develop a graphical user interface for the model that will likely be available by the time we intend to begin modelling. Stephen Price at

Los Alamos National Laboratory has gridded physical input data for Greenland for relevant factors such as glacial geometry, temperature, and basal topography. These inputs will be available for our use. If we wish to run sophisticated enough models to require advanced computing capabilities, we may use the supercomputer resources of the Vermont Advanced Computing Center or build an appropriately fast computer.

The primary objective of the modelling is to determine likely zones of basal freezing and melting during the history of the Greenland Ice Sheet. Using the ice core temperature proxy record as an input for temperature over time, we can use GLIMMER to map possible freezing and melting regions in Greenland over time. Then we will construct basal flow paths that could constrain the geographic regions where clasts are sourced, thereby constraining the minimum degree of ice sheet retreat associated with our burial histories. The general method in modelling will be to run several model experiments with different sets of physically plausible initial conditions consistent with our data. Thus we should produce a range of acceptable interpretations of the data, including a "preferred" interpretation of the data, which is produced by the physical inputs we consider to be the most likely. The modelling component is further complicated by the fact that our data may necessitate some revision of the accepted models. For example, our study is the first empirical test of the modelled result that there was no significant Eemian ice sheet retreat at the latitude of Upernavik. Should we find ~130k exposure ages at Upernavik, this would require running experimental input conditions into the model that attempt to reproduce this.

Timeline:

May-June, 2008: Preparation, including forward modelling of isotope inventories generated by possible exposure and burial histories

July, 2008: Field work in Greenland

Fall, 2008: Prepare and purify mineral isolates at UVM

Fall, 2008 and Spring 2009: Measure isotope concentrations at Livermore National Lab

Spring and Summer 2009: Begin ice modelling experiments, continue clast analyses

Fall, 2009: Start writing thesis

Spring, Summer 2010: Finish and defend thesis, academic publication if possible

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Appendix A – Equations

In this appendix I summarize the physical equations discussed in the main text that are employed in the GLIMMER model.

Glen's flow law is commonly written $\varepsilon_{xy} = A\tau^{n}{}_{xy}$

Where ε is strain, τ is stress, n is a empirically derived constant approximately equal to 3, and A is a combination of the factors of ice temperature, crystal orientation, impurity content and other factors [Patterson, 1994].

The most commonly modelled of the factors affecting A is temperature. A typical equation for temperature is $A = A_0^{-Q/RT}$

Where T is temperature, A_0 is independent of temperature, R is the universal gas constant, and Q is the experimentally determined activation energy from creep [Paterson, 1994].

The following equations are used to drive the GLIMMER model. This material is modified from the GLIMMER support website:
http://wiki.nesc.ac.uk/read/glimmer-project

The equation for ice thickness evolution is given by:

$$\frac{\partial H}{\partial t} = -\nabla \cdot (\bar{u} H) + B$$

Where H is ice thickness, t is time, $\boldsymbol{\nabla}$ is the divergence operator, $\boldsymbol{u}$ is vertically-averaged ice velocity, and B is the net surface mass balance (change in accumulation or ablation).

This version of Hutter's (1983) shallow ice approximation is used:

$$\tau_{xz}(z) = -pg(s-z)\frac{\partial s}{\partial x} \qquad \tau_{yz}(z) = -pg(s-z)\frac{\partial s}{\partial y}$$

Where *s* is the height of the ice surface, *p* is the density of ice, and *g* is the acceleration from gravity.

This version of Glen's flow law is developed:

$$\dot{\epsilon}_{iz} = \frac{1}{2}\left(\frac{\partial u_i}{\partial z} + \frac{\partial u_z}{\partial i}\right) = A(T^*)\tau_*^{(n-1)}\tau_{iz} \qquad i = x, y,$$

Using the variables described above.

The above relationship from Paterson, 1994 is used for A(T)

The time dependent ice-temperature is approximated as:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\nabla^2 T - \boldsymbol{u} \cdot \boldsymbol{\nabla} T + \frac{\Phi}{\rho c} - w\frac{\partial T}{\partial z},$$

Where T is temperature, k is the thermal conductivity of ice, c is the specific heat capacity of ice, Φ is the heat generated due to internal friction, and w is vertically averaged velocity.

Basal sliding velocity is given at zero-order by

$$u_b = t_b \tau_b$$

Where t_b is the basal traction factor and τ_b is basal shear stress.

The melting and freezing rate at the ice sheet base are then calculated by

$$\dot{B} = \frac{H_o - H_i}{\rho_{\text{ice}} L}$$

Where H_o is the outgoing heat flux from the ice sheet to the bedrock, H_i is the incoming heat flux from the bedrock to the ice sheet, and L is the latent heat of fusion of water.

The outgoing heat flux is defined by

$$H_o = -k_{ice} \frac{\partial T}{\partial z}_{z=h}$$

The incoming heat flux is given by

$$H_i = -k_{rock} \frac{\partial T}{\partial z}_{z=h^-} + \boldsymbol{u}_b \cdot \boldsymbol{\tau}_b + \begin{cases} \rho_{\text{ice}} \dot{B}/L & \text{when } \dot{B} < 0 \\ 0 & \text{otherwise} \end{cases}$$

Geothermal heat flux is provided by

$$\frac{\partial T}{\partial t} = \frac{k_{\text{rock}}}{\rho_{\text{rock}} c_{\text{rock}}} \nabla^2 T = \frac{k_{\text{rock}}}{\rho_{\text{rock}} c_{\text{rock}}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right),$$

GLIMMER solves these equations numerically, employing a series of gridded nodes and averaging values over the nodes. The model uses two staggered horizontal grids in order to improve the stability of the analysis. Additional derivations and equations are employed by GLIMMER that are not shown here.