

Math 255 - Spring 2022
The ring $\mathbb{Z}/n\mathbb{Z}$
5 points

This homework invites you to spend a little bit of time thinking about $\mathbb{Z}/n\mathbb{Z}$ as a ring, and to practice the computation of the inverse of an element modulo n , with concrete examples.

1. (a) Please give a multiplication table for the ring $\mathbb{Z}/12\mathbb{Z}$.
(b) List all units in the ring $\mathbb{Z}/12\mathbb{Z}$.
(c) List all zero divisors in the ring $\mathbb{Z}/12\mathbb{Z}$.
2. (a) It is a fact that $\gcd(7, 23) = 1$. Please give an integer solution to the equation $7x + 23y = 1$.
(b) It is also a fact that the equivalence class of 7 in $\mathbb{Z}/23\mathbb{Z}$ is a unit. Please give any representative for the class that is its multiplicative inverse. In other words, please give any integer v such that

$$7v \equiv 1 \pmod{23}.$$

Hint: Consider part (a), and in particular the whole equation modulo 23.