

Math 255 - Spring 2022
Solving $ax + by = c$ completely
30 points

This project invites you to read Section 3 of our textbook, Linear Diophantine Equations, and to solve some problems to demonstrate your understanding.

Roughly speaking, this section expands our work on solving the equation $ax + by = d$, where $d = \gcd(a, b)$. We showed how to do this using the Euclidean algorithm and back-substitution. In this section we generalize this in two ways:

1. Section 3 shows how to give **all** integer solutions to the equation $ax + by = d$, where $d = \gcd(a, b)$, from knowledge of one such solution.
2. Section 3 proves that the equation $ax + by = c$ has integer solutions if and only if $\gcd(a, b)$ divides c , and shows how to obtain all integer solutions in this case.

For this assignment, please turn in your answer to the following questions:

1. Please read Section 3 of our book, and answer the Exercises along the way. (There are five exercises in this section, numbered 1 through 5.)
2. Please use the Euclidean algorithm, back-substitution, and Theorem 1 of Section 3 to give all integer solutions to the following equations:
 - (a) $x + y = 2$
 - (b) $6x + 15y = 51$
 - (c) $15x + 16y = 17$