

Math 255 - Spring 2022  
The divisors of an integer  
15 points

This homework invites you to think more deeply about the functions  $d$  and  $\sigma$  from Section 7 of the book (the number of positive divisors and the sum of positive divisors functions). Please remember that you can always solve as many or as few problems as you would like!

1. (3 points) What is the smallest integer  $n$  such that  $d(n) = 8$ ?
2. (4 points) Show that  $d(n)$  is an odd integer if and only if  $n$  is a perfect square.
3. (4 points) Show that  $d(n) \leq 2\sqrt{n}$ .
4. (4 points) For which values of  $n$  is  $\sigma(n)$  odd?