Math 395 - Spring 2020
Homework 9

This homework is due on Monday, March 30.

All of these problems must be typed up.

1. Find the number of similarity classes of 10×10 matrices A with entries from $\mathbb{Q}$ satisfying $A^{10} = I$ but $A^i \neq I$ for $1 \leq i \leq 9$, where I is the identity matrix. (You do not need to exhibit representatives of the classes.)

2. (a) Find all possible canonical forms for a matrix over $\mathbb{F}_3$ with characteristic polynomial $x^4 - 1$.

   (b) Find all possible canonical forms for a matrix over $\mathbb{F}_2$ with characteristic polynomial $x^4 - 1$.

3. (a) How many similarity classes of 8×8 matrices A with rational entries are there that satisfy $A^8 = I$ but $A^n \neq I$ for every $n \in \{1, \ldots, 7\}$? (Justify.)

   (b) Answer the same question as in part (a) but with the field of rational numbers replaced by the field $\mathbb{F}_2$ with 2 elements. (Justify.)

4. (a) How many similarity classes of 8×8 matrices A with rational entries are there that satisfy $A^8 = A$? (Explain briefly; you need not explicitly list all classes.)

   (b) How many similarity classes of 3×3 matrices A with entries from the field $\mathbb{F}_7$ are there that satisfy $A^8 = A$? (Explain briefly; you need not explicitly list all classes.)

5. Let $\mathbb{F}_q$ be the field with q elements. Find the number of similarity classes of 5×5 matrices A over $\mathbb{F}_q$ that satisfy $A^q = I$, where I is the identity matrix. (Justify your answer. You do not need to exhibit representatives of the classes.)

6. Over the finite field $\mathbb{F}_{17}$, the polynomial $x^{10} - 1$ factors into irreducible polynomials as follows:

$$x^{10} - 1 = (x-1)(x+1)(x^4 + x^3 + x^2 + x + 1)(x^4 - x^3 + x^2 - x + 1).$$

   (a) Find, with brief justification, the number of similarity classes of 8×8 matrices A with entries from $\mathbb{F}_{17}$ that satisfy $A^{10} = I$ but $A^i \neq I$ for $1 \leq i \leq 9$.

   (b) Exhibit one explicit matrix A satisfying the conditions of (a).

   (c) What is the smallest n such that the matrix you found in (b) is similar to a diagonal matrix over the field $\mathbb{F}_{17^n}$?