

Math 395 - Spring 2020
Homework 4

This homework is due on Monday, February 10.

These problems can be turned in by hand:

Section 7.6: 1, 3

Section 8.1: 11

Section 8.2: 5

Section 8.3: 8

This problem must be typed up:

1. Let R be a Euclidean Domain with respect to the norm N . Let

$$d = \min\{N(x) \mid x \in R - \{0\}\}.$$

- (a) Prove that every nonzero element of norm d is a unit in R .
- (b) Let $R = \mathbb{Z}[\sqrt{2}]$. You may assume R is a Euclidean Domain with respect to the norm

$$N(a + b\sqrt{2}) = |a^2 - 2b^2|.$$

Prove that R contains infinitely many units. (Hint: Find a unit of infinite order.)