

Name:

**Problem 1:** Pat and Sam are studying for their final exam in Topology.

Sam presents the following theorem and proof to Pat: “Theorem: The preimage of a compact set by a continuous map is compact.

Proof: Let  $f: X \rightarrow Y$  be a continuous map, with  $Y$  compact. We show that  $X = f^{-1}(Y)$  is compact. Let  $\mathcal{A}$  be a cover of  $Y$  by open sets. Since  $Y$  is compact, there is a finite subcover. Now the inverse images of these open sets are open and cover  $X$ , and therefore  $X$  is compact.”

Pat says: “This doesn’t feel right, I thought that the **image** of a compact set by a continuous map was compact...”

Who is correct? Pat or Sam? Justify your answer briefly.