

Math 295 - Spring 2020
Homework 6
Review Homework for Exam 1

This homework is due on Wednesday, February 19.

1. Show that the dictionary order, given on page 26 of our textbook, is an order relation.
2. Let X and Y be sets, and let $\pi_1: X \times Y \rightarrow X$ be the projection map. Show that for $A \subset X$, $\pi_1(\pi_1^{-1}(A)) = A$. For $W \subset X \times Y$, decide which one of $\pi_1^{-1}(\pi_1(W))$ or W contains the other, and support your answer with a proof. Give an example to show that in general the containment is strict.
3. Let \mathcal{T} and \mathcal{T}' be two topologies on the set X . Is $\mathcal{T}'' = \mathcal{T} \cup \mathcal{T}'$ also a topology on X ?
4. Consider \mathbb{Z}_+ , the set of positive integers, as a subspace of the set \mathbb{R} with the standard topology. Show that the subspace topology \mathbb{Z}_+ inherits from \mathbb{R} is the discrete topology.
5. Let X be a set.
 - (a) Assume that X is given the finite complement topology, which is described on page 77 of our textbook. Describe the closed sets of X .
 - (b) Assume that X is given the discrete topology. Describe the closed sets of X .
6. Let X be a topological space. Show that if $A \subset B \subset X$, then $\overline{A} \subset \overline{B}$.
7. Prove that 0 is a limit point of the subset $(0, 1] \subset \mathbb{R}$, where \mathbb{R} is given the standard topology.
8. Let X be a topological space. Consider the space $X \times X$ with the product topology, and its subset
$$\Delta = \{x \times x \in X \times X \mid x \in X\}$$
called the *diagonal of X* . Suppose that Δ is closed in $X \times X$. Show that X is Hausdorff.
9. Let X be a topological space and Y be a subspace of X . Show that the inclusion map $i: Y \rightarrow X$ is continuous.