Math 295 - Spring 2020
Homework 4

This homework is due on Wednesday, February 5. All problems are adapted from Munkres's *Topology*.

1. Show that if Y is a subspace of X, and A is a subset of Y, then the topology A inherits as a subspace of Y is the same as the topology it inherits as a subspace of X.

2. Consider the set $Y = [-1, 1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$?

   (a) $A = \{x \mid \frac{1}{2} < |x| < 1\}$

   (b) $B = \{x \mid \frac{1}{2} < |x| \leq 1\}$

   (c) $C = \{x \mid \frac{1}{2} \leq |x| < 1\}$

   (d) $D = \{x \mid \frac{1}{2} \leq |x| \leq 1\}$

   (e) $E = \{x \mid \frac{1}{2} < |x| < 1 \text{ and } 1/x \notin \mathbb{Z}_+\}$

3. A map $f\colon X \to Y$ is said to be an **open map** if for every open set U of X, the set $f(U)$ is open in Y. Show that $\pi_1\colon X \times Y \to X$ and $\pi_2\colon X \times Y \to Y$ are open maps.

4. Show that the dictionary order topology on the set $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ with the discrete topology.

Extra problems for graduate credit:

1. Show that the countable collection

$$\{(a, b) \times (c, d) \mid a < b \text{ and } c < d, \text{ and } a, b, c, d \text{ are rational numbers}\}$$

   is a basis for the standard topology on $\mathbb{R}^2$.

2. Consider the topology $\mathbb{R}_\ell$ on the set $\mathbb{R}$ which is described in Section 13 of the book (page 82). If L is a straight line in the plane, describe the topology that L inherits as a subspace of $\mathbb{R}_\ell \times \mathbb{R}$ and as a subspace of $\mathbb{R}_\ell \times \mathbb{R}_\ell$. Hint: In each case it is a familiar topology.