Math 295 - Spring 2020
Homework 2

This homework is due on Wednesday, January 22. All problems are adapted from Munkres's *Topology*.

1. For this problem, please assume that the real line has the least upper bound property.
   (a) Show that the set $[0, 1] = \{x \mid 0 \leq x \leq 1\}$ has the least upper bound property.
   (b) Show that the set $[0, 1) = \{x \mid 0 \leq x < 1\}$ has the least upper bound property.
2. Prove the following two "laws of algebra" for $\mathbb{R}$, using only axioms (1)-(5) on pages 30-31 of the book:
   (a) If $x + y = x$, then $y = 0$.
   (b) $(-1) \cdot x = -x$ for all x
3. Prove the following two "laws of inequalities" for $\mathbb{R}$, using axioms (1)-(6) on pages 30-31 of the book:
   (a) $x > y \iff -x < -y$
   (b) $x > y$ and $z < 0$ implies $xz < yz$.
4. If $A \times B$ is finite, does it follow that A and B are finite? Prove or give a counter-example.