Math 295 - Spring 2020
Homework 13

This homework is due on Wednesday, April 15. This problem is adapted from Munkres's *Topology*. You are welcome to search the internet for proofs/counterexamples as usual, but please cite your source!

1. (a) Let $X$ and $Y$ be two path connected spaces. Is $X \times Y$ necessarily path connected? Either prove that it is or give a counterexample.

   (b) Let $A \subset X$ be a path connected subspace of $X$. Is $\bar{A}$ necessarily path connected? Either prove that it is or give a counterexample.

   (c) If $f \colon X \to Y$ is continuous and $X$ is path connected, is $f(X)$ necessarily path connected? Either prove that it is or give a counterexample.

   (d) If $\{A_\alpha\}$ is a collection of path connected subspaces of $X$ and if $\bigcap A_\alpha \neq \varnothing$, is $\bigcup A_\alpha$ necessarily path connected? Either prove that it is or give a counterexample.