Math 295 - Spring 2020
Homework 1

This homework is due on Friday, January 17. All problems are adapted from Munkres's *Topology*.

1. Write the contrapositive and converse of the statement: "If $x < 0$, then $x^2 - x > 0$."
2. Let A and B be sets of real numbers. Write the negation of the following two statements:
   (a) For every $a \in A$, it is true that $a^2 \in B$.
   (b) For at least one $a \in A$, it is true that $a^2 \in B$.
3. Let $\mathcal{A}$ be a nonempty collection of sets. Determine the truth of each of the following statements and of their converses:
   (a) $x \in \cup_{A \in \mathcal{A}} A \implies x \in A$ for at least one $A \in \mathcal{A}$
   (b) $x \in \cup_{A \in \mathcal{A}} A \implies x \in A$ for every $A \in \mathcal{A}$
   (c) $x \in \cap_{A \in \mathcal{A}} A \implies x \in A$ for at least one $A \in \mathcal{A}$
   (d) $x \in \cap_{A \in \mathcal{A}} A \implies x \in A$ for every $A \in \mathcal{A}$
4. Let $f\colon A \to B$, $A_0 \subset A$, and $B_0 \subset B$.
   (a) Show that $A_0 \subset f^{-1}(f(A_0))$ and that equality holds if f is injective.
   (b) Show that $f(f^{-1}(B_0)) \subset B_0$ and that equality holds if f is surjective.
5. Consider the following relation $<_N$ on $\mathbb{R}$:
$$x <_N y \quad \text{if } x^2 < y^2 \text{ or if } x^2 = y^2 \text{ and } x < y,$$
   where $<$ is the usual order relation on $\mathbb{R}$. Show that $<_N$ is an order relation.