Math 395 - Fall 2021
Beginner Reading 6

This reading is "due" on Monday October 11 at 11:59pm.

This week you are invited to read Chapter 5 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 5.1

1. True or false: Let $G = G_1 \times G_2$ and define the elements $g = (g_1, 1) \in G$ and $h = (1, h_1) \in G$. Then $gh = hg$.

2. What is an elementary abelian group?

3. How many subgroups of size p are there in the elementary abelian group of size p^2?

Section 5.2

4. What is the free abelian group of rank r?

5. Can the numbers $n_1 = 7, n_2 = 6, n_3 = 3$ be the invariant factors of a finite group G?

6. List all of the possible sets of invariant factors for an abelian group G of order 45.

7. True or false: The elementary divisors of G are the invariant factors of the Sylow subgroups of G.

8. What is the primary decomposition theorem for finite abelian groups?

9. Now let G be a finite abelian group. What is defined to be the rank of G? How is that different from the free rank?

10. What is the exponent of a group G?

Section 5.3: No questions on section 5.3 but I recommend you familiarize yourself with the table of groups of small order; it's super handy to know!

Section 5.4

11. If G is a group, what is its commutator subgroup? How is it denoted?

12. True or false: The commutator subgroup is the set of all commutators of the group G.

13. What does the recognition theorem say?

Section 5.5

14. To write the direct product of two groups H and K, all you need are the binary operations on H and K. To write a semidirect product you need an extra piece of data. What is it?

15. How can you tell if a semidirect product is just a direct product?

16. What is a complement for a subgroup H of a group G?