Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Friday: Quiz 2 on HW2

↳ a selection of problems from HW 2

choose one + solve it

Quiz can be handwritten

Final copy of HW on Gradescope due Friday

#ScholarStrike

scholarstrike.com

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#4 $\varphi: S_3 \hookrightarrow S_6$

$$S_3 = \langle (12), (123) \rangle$$

$$\begin{aligned}
1 &\longmapsto 1 \\
(12) &\longmapsto (12) \leftarrow \\
(13) &\longmapsto (13) \\
(23) &\longmapsto (23) \\
(123) &\longmapsto (123) \leftarrow \\
(132) &\longmapsto (132)
\end{aligned}$$

$$= (123)(123)$$

$(12)(123) = (23)$

$(123)(12) = (13)$

$(12)(123)[1]$

feeding 1 into the permutation

$$\begin{aligned}\varphi((23)) &= \varphi((12)(123))\\ &= \varphi((12))\cdot\varphi((123))\\ &= (12)(123)\\ &= (23)\end{aligned}$$

$= (12)[2]$

$= [1]$

. . . .

$S_n = \langle (12), (123\ldots n) \rangle$

$\uparrow$

A_n

Conjecture: if $\varphi: G \hookrightarrow S_n$

and (12) & $(123\ldots n) \in \mathrm{Im}\varphi$

then $G = S_n$

Prob not actually

Definition of left regular representation:

- Every group acts on itself by left multiplication

$$G \curvearrowright G$$

$g \in G$ (group)

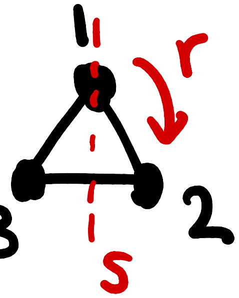
$\alpha \in G$ (set)

$g \cdot \alpha = g\alpha$ ← multiplication in gp

- $G \curvearrowright A$ (A is a set with n elements)

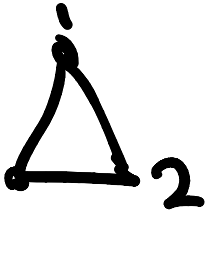
$$\Updownarrow \quad \text{via numbering the elements of } A$$

$\varphi: G \to S_n$ gp hom

Example: $D_3 \curvearrowright$ 

(vertices $= A$)

$$\varphi: D_3 \to S_3$$
$$r \mapsto (123)$$

$D_3 \times D_2 \curvearrowright$ 

not a set with n elements

$M_{2\times 2}(\mathbb{R}) \curvearrowright \mathbb{C} \cup \{\infty\}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot x = \frac{ax+b}{cx+d}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot x = \frac{-x}{-1} = x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot x$$

Left regular representation of G is the specific gp hom $\varphi: G \to S_n$ $(n = \# G)$ obtained by having G act on itself on the left.

Example $C_3 = \langle g \rangle = \{1, g, g^2\}$

$\phantom{C_3 = \langle g \rangle = \{}1 \quad 2 \quad 3$

Get a map $\boxed{\varphi : C_3 \to S_3}$

$$1 \mapsto 1$$
$$g \mapsto (123)$$
$$g^2 \mapsto (132)$$

this hom is "the" left regular rep of C_3

$g \cdot 1 = g \qquad 1 \mapsto 2$

$g \cdot g = g^2 \qquad 2 \mapsto 3$

$g \cdot g^2 = 1 \qquad 3 \mapsto 1$

$g^2 \cdot 1 = g^2 \quad (1 = 3) \qquad 1 \mapsto 3$

$g^2 \cdot g = 1 \quad (2 = 1) \qquad 2 \mapsto 1$

$g^2 \cdot g^2 = g \quad (3 = 2) \qquad 3 \mapsto 2$

That’s all for today!