Math 395 - Fall 2020

Quiz 8

Please solve **ONE** of the three problems below:

1. Let $K = \mathbb{Q}(\sqrt{3+\sqrt{5}})$.
   (a) Show that $K/\mathbb{Q}$ is a Galois extension.
   (b) Determine the Galois group of $K/\mathbb{Q}$.
   (c) Find all subfields of K.

2. Let K be the splitting field of $(x^2-3)(x^3-5)$ over $\mathbb{Q}$.
   (a) Find the degree of K over $\mathbb{Q}$.
   (b) Find the isomorphism type of the Galois group $\mathrm{Gal}(K/\mathbb{Q})$.
   (c) Find, with justification, all subfields F of K such that $[F : \mathbb{Q}] = 2$.

3. Let $\alpha = \sqrt{1-\sqrt[3]{5}} \in \mathbb{C}$ (where $\sqrt[3]{5}$ denotes the real cube root), let K be the splitting field of the minimal polynomial of α over $\mathbb{Q}$, and let $G = \mathrm{Gal}(K/\mathbb{Q})$.
   (a) Find the degree of $\mathbb{Q}(\alpha)$ over $\mathbb{Q}$.
   (b) Show that K contains the splitting field of x^3-5 over $\mathbb{Q}$ and deduce that G has a normal subgroup H such that $G/H \cong S_3$.
   (c) Show that the order of the subgroup H in (b) divides 8.