Math 395 - Fall 2020
Quiz 2

Please solve **ONE** of the two problems below:

1. Let G be a finite group acting transitively (on the left) on a nonempty set Ω. For $\omega \in \Omega$, let G_ω be the usual stabilizer of the point ω:

$$G_\omega = \{g \in G : g\omega = \omega\},$$

where $g\omega$ denotes the action of the group element g on the point ω.

   (a) Prove that $hG_\omega h^{-1} = G_{h\omega}$ for every $h \in G$.

   (b) Assume that G is abelian. Let N be the kernel of the transitive action. Prove that $N = G_\omega$ for every $\omega \in \Omega$.

   (c) Show that part (b) is not true if G is not abelian. In other words, give an example of a finite group G and a nonempty set Ω on which G acts transitively on the left such that $N \neq G_\omega$ for some ω.

2. Let G be a group and let H be a subgroup of finite index $n > 1$ in G. Let G act by left multiplication on the set of all left cosets of H in G.

   (a) Prove that this action is transitive.

   (b) Find the stabilizer in G of the identity coset $1H$.

   (c) Prove that if G is an infinite group, then it is not a simple group.