Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Today + wednesday : ask me anything

Friday: Whole class period is midterm

2 new questions on gp theory

#2a) $\#G = 10{,}989 = 3^3 \cdot 11 \cdot 37$

$n_3 \mid 11 \cdot 37$

$n_3 \equiv 1 \bmod 3$

$m \mid 11 \cdot 37$, $m = 11^i \cdot 37^j$ $i = 0, 1$, $j = 0, 1$

$1, \cancel{11}, \underbrace{37}_{36+1}, \cancel{11 \cdot 37}$ ← all the divisors of $11 \cdot 37$

$\cancel{11 \cdot 37} \equiv 2 \cdot 1 \equiv 2 \bmod 3$

$n_3 = 1$ OR 37

$n_3 = \dfrac{\#G}{\#N_G(P_3)} = [G : N_G(P_3)]$

$\Rightarrow \#N_G(P_3) = \dfrac{\#G}{n_3}$

$n_3 = 37$, $\#N_G(P_3) = 3^3 \cdot 11$

if $n_3 = 1$, $\#N_G(P_3) = 3^3 \cdot 11 \cdot 37$

$n_{11} \mid 3^3 \cdot 37$ all divisors of $3^3 \cdot 37$ are of the form

$n_{11} \equiv 1 \bmod 11$

$3^i \cdot 37^j$ $\quad i = 0, 1, 2, 3$

$j = 0, 1$

$j=0 \quad 1, \cancel{3}, \cancel{3^2}, \cancel{3^3}$ $\quad 22+5$

$j=1 \quad \cancel{37}, 3\cdot 37, \cancel{3^2\cdot 37}, \cancel{3^3\cdot 37}$

$33+4$

$3\cdot 37 \equiv 3\cdot 4 \bmod 11$

$3^3\cdot 37 \equiv 3\cdot 3 \equiv 9 \bmod 11$

$3^2\cdot 37 \equiv 3(3\cdot 37) \equiv 3\cdot 1 \bmod 11$

$n_{11} = 1 \quad \# N_G(P_{11}) = 3^3 \cdot 11 \cdot 37$ OR $n_{11} = 3 \cdot 37$

$\# N_G(P_{11}) = 3^2 \cdot 11$

$n_{37} \mid 3^3 \cdot 11$

$n_{37} \equiv 1 \bmod 37$

$n_{37} = 1,\ 3^3 \cdot 11$

$\# N_G(P_{37}) = 3^3 \cdot 11 \cdot 37$

$\# N_G(P_{37}) = 37$

b) $n_{37} = 1,\ 3^3 \cdot 11$

$n_3 = 1, 37$

If the Sylow 37-subgp is normal done, so suppose not.

$\vdots$

Can conclude that $n_3 = 1$, done

$n_{37} = 1, 3^3 \cdot 11$ 1^{st} thing to try is counting. Here any

$n_3 = 1, 37$ $\# P_{37} = 37$ which is a prime

$\Rightarrow P_{37} \cong C_{37}$ so any 2 distinct Sylow 37-sub

must intersect in only the identity

Each 37-Sylow has 36 elements of order 37

$3^3 \cdot 11 \cdot 36$ elements of order 37 in G

Sylows

If p is a prime

- $\#G = p \Rightarrow G \cong C_p$ cyclic gp with p elements

- $\#G = p$ in G 1 identity ← only element of order 1
  $\underbrace{p-1}_{\text{the rest}}$ elements have order p

because $\operatorname{order}(g) \mid \#G = p$

$\# G = 3^3 \cdot 11 \cdot 37$

$\#$ elts of order $37 = 3^3 \cdot 11 \cdot 36$

left over are

$3^3 \cdot 11 \cdot 37 - 3^3 \cdot 11 \cdot 36$

$= 3^3 \cdot 11 \, (37 - 36)$

$= 3^3 \cdot 11$

elements not of order 37

For a contradiction, assume we have 37 Sylow 3-subgps

$\# P_3 = 3^3$

Let $P, Q \in Syl_3(G)$

$\# P \cap Q = 1$

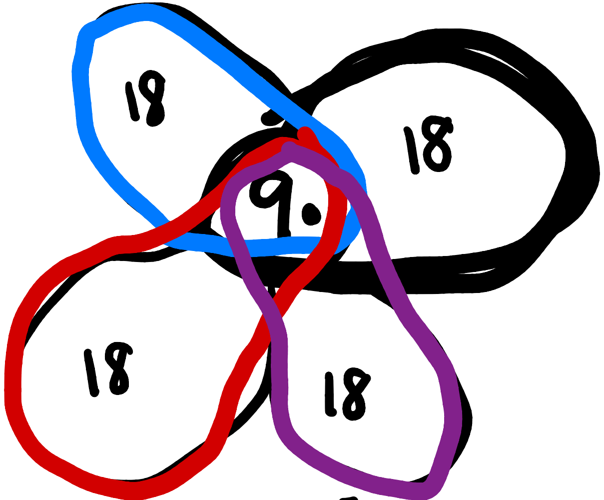
$\# P \cap Q = 3$

$\# P \cap Q = 9$

or $P = Q$

If I have 37 Sylow 3-subgps what is the minimum number of elements that have order a power of 3?

↳ equivalent to saying are contained in a Sylow 3-subgp.



Then there are

$37 \cdot 18 + 9$

elements of order a power of 3 (this includes identity)

Is $37 \cdot 18 + 9 > 3^3 \cdot 11$?

$$\begin{array}{ccc} & & 3^3 \cdot 11 \\ & & \shortparallel \\ 3^2(37 \cdot 2 + 1) & & 3^2(3 \cdot 11) \\ 3^2 \cdot 75 & > & 3^2 \cdot 33 \end{array}$$

yes

$$\begin{array}{r} 37 \\ 18 \\ \hline \end{array}$$

Not enough "elements left over" to have both $3^3 \cdot 11$ Sylow 37-subgps ($3^3 \cdot 11 \cdot 36$ elts) and 37 Sylow 3-subgps (at minimum $37 \cdot 18 + 9$ elts)

3-Sylows contain only elements of order a power of 3

37-Sylows contain only elements of order a power of 37

if $P \in Syl_p(G)$ $Q \in Syl_q(G)$ $p \neq q$ primes

$$P \cap Q = 1$$

I will write some remarks on 2c)

on Campuswire

FOR OH this week please send me a meeting request Tuesday 1pm-3pm
Thursday all day BUT 3pm-4pm

That's all for today!

OH canceled today